A METHOD OF OPTIMIZING THE EFFECTIVENESS OF AN "OVERALL WEAPONS SYSTEM" WITH RESPECT TO IT'S RECONNAISSANCE SUB-SYSTEM

ARTHUR KING BENNETT, JR.

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by

ARTHUR KING BENNETT, Jr.
Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of

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Monterey, California

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PREFACE

The method of optimizing the effectiveness of an "overall weapons system" and the concept of an overall system in this paper, stemmed from work carried on at the Military Operations Research Group of Lockheed Aircraft Corporation during the 1953 summer field trip. The models and examples used reflect operational and combat experience in the reconnaissance field as closely as feasibility of solution and security will allow.

The writer would like to express his appreciation to Associate Professor S.H. Kalmbach and Professor C.C. Torrance of the United States Naval Postgraduate School staff for their help as advisors and to R.A. Bailey and H.P. Gruner, Jr. of Lockheed Aircraft Corporation for their aid in formulating the problem.

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TABLE OF SYMBOLS AND ABBREVIATIONS

M_m	the class of all missions
W_w	the class of all weapons sub-systems
L_L	the class of all reconnaissance sub-systems
N_n	the class of all targets
Z_z	all values of information deterioration
R_r	all responses of decisioner
P_K	probability of a kill by overall weapons system per action
P_c	probability of a contact of $\underline{n}$ by $\underline{w}$
P_{KIC}	probability of a kill of $\underline{n}$ by $\underline{w}$ given a contact
C_w	cost of weapons sub-system per action
N_L^w	ratio of weapons sub-system actions to reconnaissance sub-system actions
C_L	cost of reconnaissance sub-system per action
C_d	cost of decision per action of $\underline{w}$
C_c	cost of communications per action of $\underline{w}$
S_L	sweep rate of reconnaissance sub-system
W	sweep width
V	sweep velocity
(xyz)	target density
μ	mean value of target distribution
σ^2	variance of target distribution
σ	standard deviation of target distribution
D	diffusion constant
c	drift constant
t_s	sweep time

t_r	return time
t_p	processing time
t_i	interpretation time
t_q	communication time
t_c	decision time
t_o	ordering time
t_b	preparation time
t_a	action time
t_d	information deterioration time
P_f	probability of weapons, sub-system finding target if in its locality
P_s	probability of weapons sub-system searching in locality of a target

CHAPTER I

INTRODUCTION

1. Summary

This paper defines an "overall weapons system" as any combination of a weapons sub-system, a reconnaissance sub-system and a decision making component joined by a communications net. This definition is shown to cover the entire range of military striking forces.

A general measure of "overall weapons system" effectiveness is defined in terms of kills per cost. A variable Z, information deterioration, which is a function of time, for a given "overall weapons system" and operating situation is also defined. The kill probability and the cost of the "overall weapons system", in the given operating situation, are both shown to be dependent upon Z, information deterioration. Kill probability and cost both increase as information deterioration is decreased. By finding kill probability and "overall weapons system" cost as functions of information deterioration when the reconnaissance sub-system is varied, the value of information deterioration corresponding to maximum effectiveness defines an optimal reconnaissance sub-system.

Similarly, proposed "overall weapons systems" may be tested against various operating situations to determine conditions under which each may be profitably used.

A carrier task force with attack aircraft and photographic reconnaissance is developed as an example (with assumed data) to show how the details of solution are worked out.

The appendix includes work on photographic sweep rates.

2. Statement of the problem

Operations Research studies of weapons systems have seldom considered the effect of intelligence, and especially reconnaissance, on the effectiveness of the "overall weapons system" of which a weapon is the major sub-system.

Though it is possible to conceive of an "overall weapons system" in which a reconnaissance sub-system is not important, this is not the usual case.

It is the purpose of this paper to define the "overall weapons system", to derive a measure of effectiveness for the "overall weapons system", and to suggest a method of optimizing the effectiveness of the "overall weapons system" with respect to its reconnaissance sub-system.

3. The "Overall Weapons System"

In the armed services a weapons system is the means to carry out a combat mission. Generally in steady state combat operations, a mission as transmitted to the operating forces contains little uncertainty as to the nature of enemy forces. The operating forces are aware of their own weapons capabilities. There usually exists, however, some uncertainty as to the location and identity of a military objective or its components. It is this location uncertainty that makes necessary the concept of an "overall weapons system" within which a reconnaissance sub-system, a decision mechanism and a communications net have been integrated with a weapon. We have assumed that such information as own equipment characteristics and enemy target characteristics are supplied by other means than the reconnaissance sub-system. Obviously this assumption is not completely true, but is a useful approximation. We will consider any

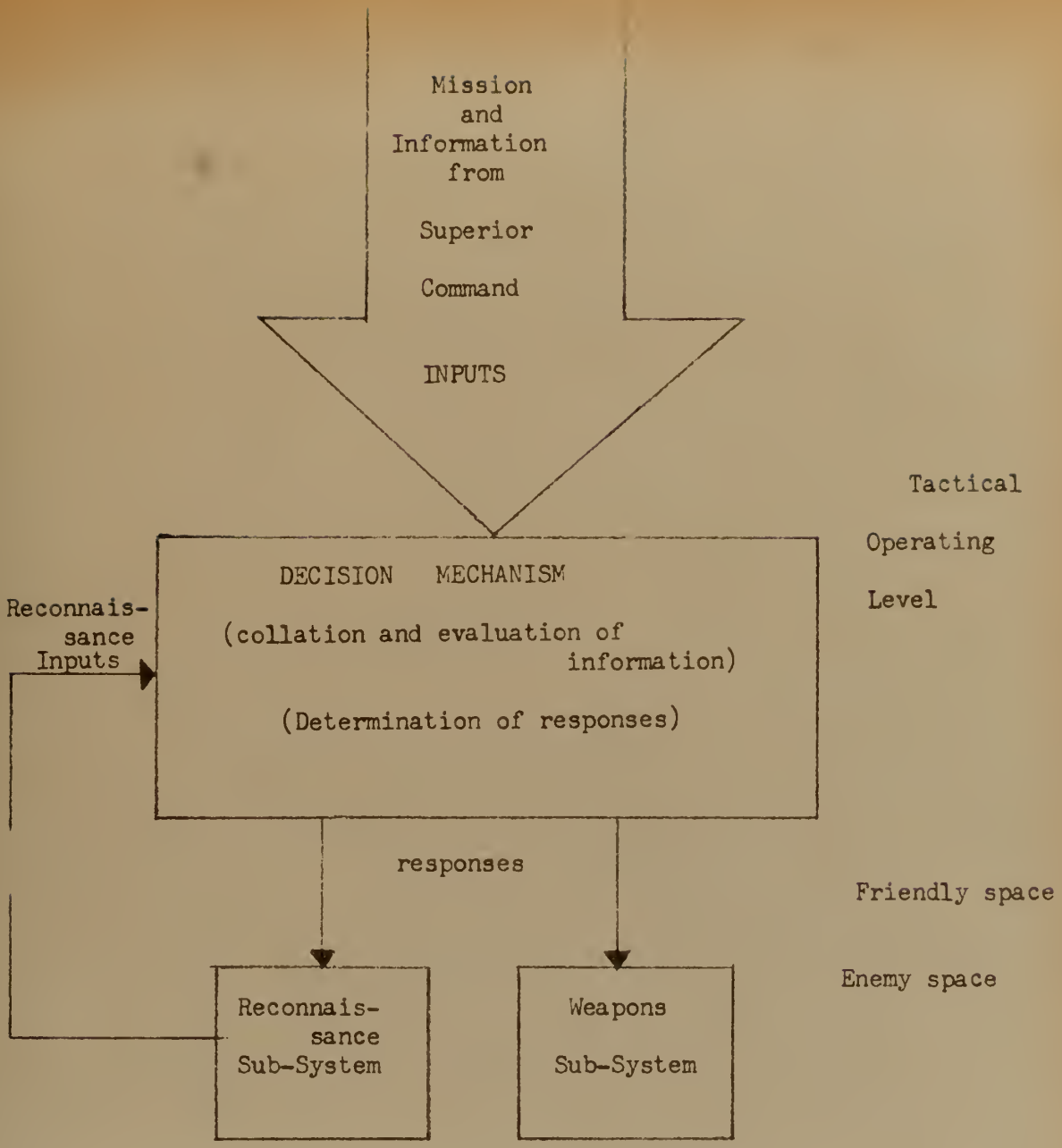


Illustration 1

Overall Weapons System

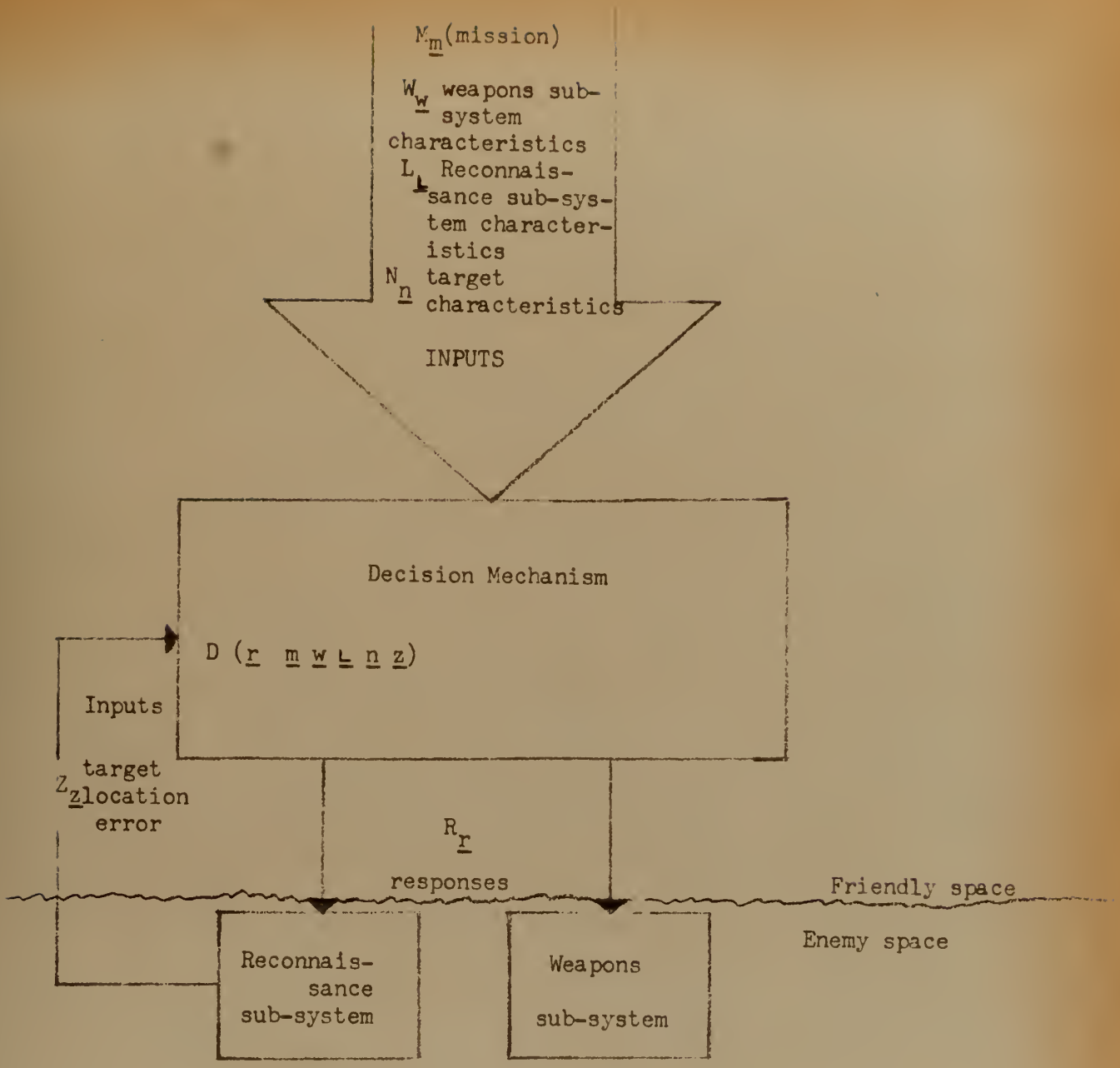


Illustration 2

Overall Weapons System

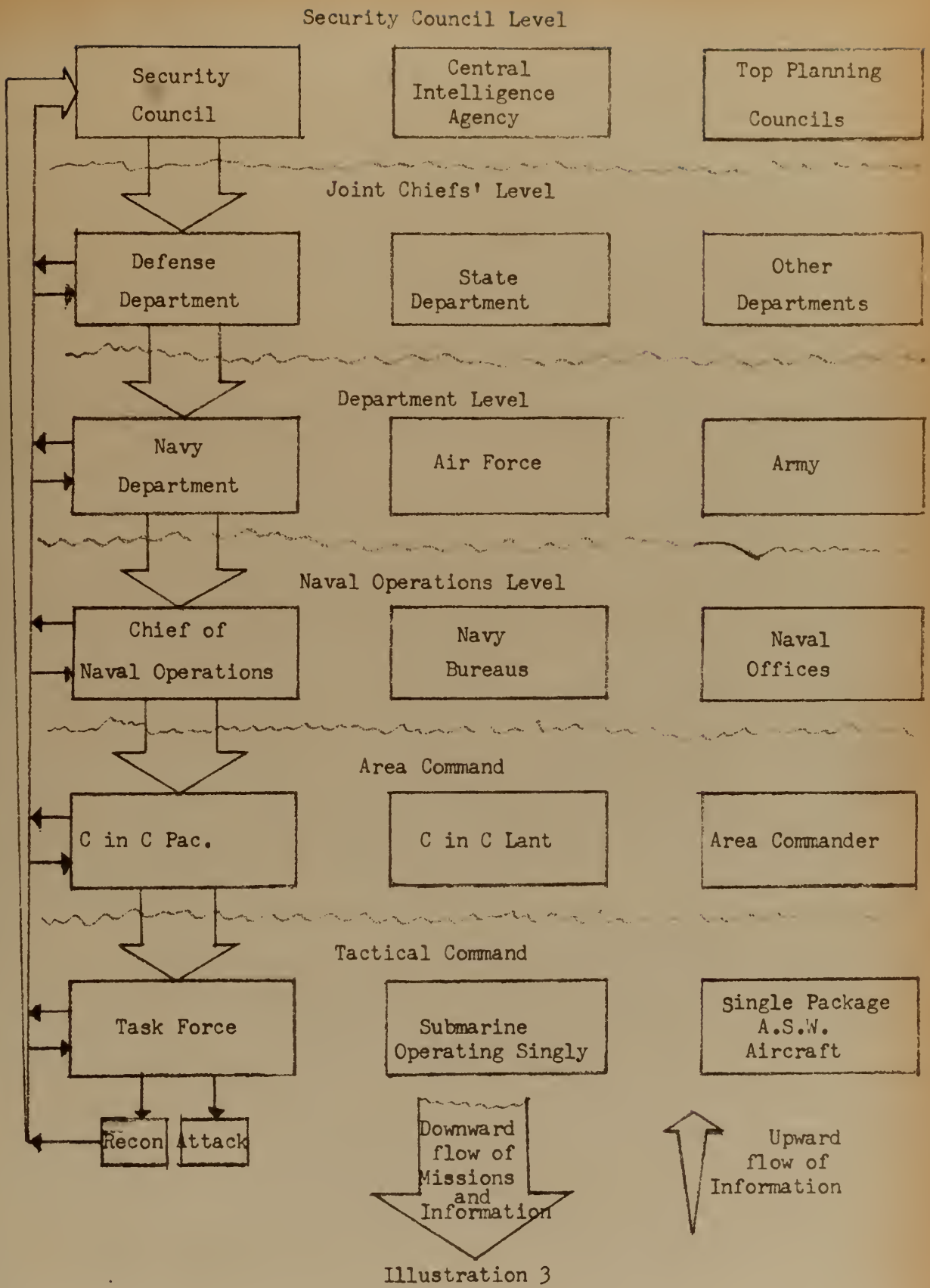


Illustration 3

Structure Providing Inputs to "overall weapons system"

information concerning enemy weapons characteristics or enemy strategy and tactics as a bonus, and will assume that a reconnaissance sub-system exists primarily to reduce the uncertainty of target locations and thereby to increase the probability that a weapons sub-system will engage a target.

When we introduce uncertainty of object location, it is necessary to expand the scope of our systems analysis. We must include a decisioner, a reconnaissance sub-system and a communications net as well as the weapons sub-system itself. The decisioner chooses between using or not using the weapons sub-system against targets located by the reconnaissance sub-system. We will define an "overall weapons system" as the military system formed by integrating a weapons sub-system, a reconnaissance sub-system, and a decision mechanism all joined by a communications net. This overall system is able to gather and receive intelligence and react logically to it.

Illustration 1 shows qualitatively how inputs to the decisioner from a higher military level, represented by the broad arrow, and from the reconnaissance sub-system are combined and result in orders to the reconnaissance and weapons sub-systems.

In illustration 2, symbols representing inputs and outputs are assigned.

The relationship shown in illustrations 1 and 2 can be extended up the national chain of command as shown in illustration 3. At each superior level the inputs to the decisioner are the same. They are:

(a) Mission

(b) Weapons sub-system characteristics

(c) Reconnaissance sub-system characteristics

(d) Enemy characteristics

(e) Uncertainty of location of military objectives

However, the scope of each input is broader at each higher level. Here we note that information which is gathered at the operating level has a large influence upon the decisions made there, and a smaller effect on decisions made by the next echelon. This effect at the higher level is reflected back down to the operating force through the responses of its decision mechanism. These responses appear as inputs to the operating level. So the effects of information continue to be felt up and down the command pyramid, but diminishing rapidly as levels removed from the operating forces are reached.

CHAPTER II

OPTIMAL "OVERALL WEAPONS SYSTEM" RESPONSES

1. Simplification of "overall weapons system"

Decisions of the "overall weapons system", as outlined in Chapter I, are affected by both the upward flow of information and the downward flow of information and decisions, phrased as missions. The effect of these interactions appears ultimately as a change of mission at the operating level. We may therefore lump all of the superior levels into a single unit which supplies information and a mission to the "overall weapons system". We may also assume that the effect of reconnaissance information is felt mainly at the operating level. We therefore have reduced our "overall weapons system" to the single stage type shown in illustrations 1 and 2. Actual "overall weapons systems" may have one or more weapons and reconnaissance sub-systems. Only those with one of each type sub-system will be considered here.

Overall weapons systems in practice will consist of:

(a) An officer in tactical command plus his staff as the decision mechanism.

(b) Reconnaissance units, craft, or equipment as the reconnaissance sub-system.

(c) Attack units, craft, or equipment as the weapons sub-system.

(d) A suitable collection of communications channels as the linking communications net.

Illustration 4 tabulates several examples of "overall weapons systems" broken down into components.

Table of components for several examples of "overall weapons systems"

Overall Weapons System	Decision Mechanism	Weapons sub-system	Reconnaissance sub-system	Elements of Communications Net
Fast Carrier Task Force	Task Force Commander and staff	Attack and Fighter Aircraft	Photo-Reconnaissance Aircraft	Radio Signals Visual Signals Oral messages Written messages Physical transport
Single Package A.S.W. Aircraft	Patrol Plane Commander and Crew	Torpedoes Bombs Depth Charges Guns	Radar Visual M.A.D.	ICS Visual Signals Oral messages Electronic Repeaters
Submarine Operating Singly	Submarine Commander and staff	Torpedoes Guns	Visual Periscope Radar E.C.M.	ICS Visual Signals Oral messages Electronic Repeaters

Illustration 4

Once we have defined the inputs received through the broad arrow of illustration 2, the average characteristics of the "overall weapons system" are defined. We now have to devise a measure of effectiveness before we can optimize the overall weapons system with respect to one or all of its components. In this paper we are interested in optimizing the "overall weapons system" with respect to its reconnaissance sub-system.

2. The elements of the simplest general problem

The simplest problem to analyze is one in which we initially have the following inputs:

(a) A single mission m which will

(1) Single out a particular class of target n as the sole type

to be operated on by the "overall weapons system".

(2) Specify a single optimization criterion (such as cost to U.S. in time, materials, etc.) with respect to which the "overall weapons system" operations on targets of class n are to be optimized.

(b) A single weapons sub-system w about which all operational characteristics are known.

(c) A single reconnaissance sub-system L about which all operational characteristics are known.

(d) The characteristics of the target type n singled out by mission m, including its density distribution in operating space, are known.

The remaining input z is the mean position error for targets of class n caused by time delays within the "overall weapons system" and the characteristics of target class n. We may define z as the information deterioration factor. It is important to note that z is a function of time and target characteristics alone, and represents the degree of deterioration of information due to target motion after the time of observation.

3. The measure of "overall weapons system" effectiveness

Suppose that mission m specified target n and dollar cost to the United States as the optimization criterion for the "overall weapons system".

Then we will use as our measure of effectiveness the following:

$$M.E. = \frac{\text{Probability of kill of target n by weapons Sub-system w with z information deterioration}}{\$ \text{ Cost of overall weapons system to U.S., pro-rated.}}$$

$$M.E. = \frac{P_K \text{ per action}}{\$ \text{ Cost to U.S. per action}}$$

4. Kill Probability

The kill probability P_K per action can be derived in the following steps:

(a) Determine the time delay t_d in the "overall weapons system", between the time of detection of the average target n by the reconnaissance sub-system L and the time weapons sub-system w can attack n.

(b) From time delay t_d and the characteristics of target n, determine z.

(c) From the characteristics of w and n and the value of z determine a probability, P_C that weapon w will contact target n.

(d) From the characteristics of w and n determine a probability P_{KIC} that weapon w will kill n given a contact on n.

(e) P_K, the probability of a kill of n by w. $P_K = P_{KIC} \cdot P_C$

5. Cost to the United States

The cost to the United States in whatever value units are specified in the "overall weapons system" mission, is the sum of the costs of the components of the "overall weapons system". These costs are:

(a) C_w, cost of weapons sub-system per action consisting of

(1) Original cost of weapon, pro-rated per action.

(2) Cost of training, maintenance and special equipment pro-rated per action.

(3) Special costs peculiar to the weapons sub-system pro-rated per action.

(b) C_L/N_w, cost of reconnaissance sub-system action, per action of $\underline{w}$,

(1) Cost of reconnaissance vehicle-medium pro-rated per reconnaissance action, divided by $N_{w)}$, the reconnaissance ratio. This cost is not applicable if the reconnaissance sub-system is carried by the same vehicle as is the weapons sub-system.

(2) Costs of training, special equipment, materials and maintenance pro-rated per action of reconnaissance sub-system, divided by $N_{(w)}$, the reconnaissance ratio.

(3) Costs of processing and interpreting reconnaissance information computed as above.

(c) C_d, cost of Decision Mechanism per action of $\underline{w}$. Cost of maintaining decisioner and staff pro-rated per action of weapons sub-system.

6. Detailed measure of effectiveness

Expanding the measure of effectiveness in terms of the probabilities and costs outlined in Sections 4 and 5 above, we get

$$M.E. = \frac{P_c \cdot P_{KIC}}{C_w \quad C/N_{(w)} \quad C_d \quad C_c}$$

$$M.E. = \frac{\text{Probability of a contact of n by w times Probability of a kill of n by w per contact}}{\text{Cost of (Weapons, Reconnaissance, Decision and Communication) Sub-systems pro-rated per weapons sub-system action}}$$

7. Use of the measure of effectiveness to determine the responses of the "overall weapons system" to a given situation

For a situation in which the following factors in the measure of effectiveness are fixed:

$$P(KIC),\ C_w,\ C_l,\ Cd\ \ C_c$$

Two values of P_c, the probability of contact, are of interest.

(a) $P_{c(lw)}$ when carrying on reconnaissance at the optimum rate determined by the reconnaissance ratio

$$N_L^W \qquad \frac{\text{weapons sub-system action}}{\text{reconnaissance sub-system action}}$$

a constant in a given situation.

(b) $P_{c(w)}$ when no separate reconnaissance is carried on. This value of P_c is a function of the weapons sub-system $\underline{w}$ and the target density.

We can now calculate two values of the measure of effectiveness corresponding to the two basic alternative responses that can be made by the decision mechanism

(a) Carry on reconnaissance and attack operations in the proper ratio.

(b) Carry on attack operations only.

$$\text{ME (reconnaissance and attack)} = \frac{P_{c(Lw)} \cdot P(KIC)}{C_{\underline{w}} \quad \frac{CL}{N_L^{(w)}} \quad C\underline{d} \quad C_c}$$

$$\text{M.E. 'attack only)} = \frac{P_{c(w)} \cdot P(KIC)}{C_w \quad Cd \quad C_c}$$

The maximum of these two values will determine the responses of the decision mechanism and thereby the action of the "overall weapons system".

CHAPTER III

RECONNAISSANCE CONSIDERATIONS

1. Reconnaissance definitions

Reconnaissance is the examination of land, sea and air space to gather information concerning the enemy.

Reconnaissance vehicle mediums are vehicles equipped with one or more reconnaissance mediums. Possible vehicles are:

(a) conventional aircraft

(b) guided missiles

(c) surface ships

(d) submarines

(e) land surface vehicles

(f) humans

Reconnaissance mediums are the physical means of absorbing information into the "overall weapons system". A few mediums are:

(a) Human lookout using eyes or, eyes plus optics, to absorb information.

(b) Human photo interpreter using eyes or, eyes plus optical equipment, to absorb information in images swept up by photographic equipment.

(c) Human radar operator, absorbing information in images swept up by radar equipment.

(d) Human communications monitor, absorbing information from visual and aural images swept up by communications receiving equipment.

(e) Human M.A.D. operator, absorbing information from visual images

swept up by M.A.D. equipment.

(f) Human E.C.M. operator, absorbing information from visual and aural images swept up by E.C.M. equipment.

The reconnaissance medium will be thought of here as a combination of the human and his equipment. Each reconnaissance medium is capable of absorbing one or more classes of reconnaissance information.

Typical reconnaissance information is:

(a) Object location.

(b) Object identification.

(c) Object characteristics.

In simplifying the reconnaissance problem, we will limit the definition of information to identification and location of objects. This limitation is in accordance with our simplifying assumption that target characteristics are given to the "overall weapons system" from higher echelons.

2. Sweep rate and reconnaissance ratio

Each reconnaissance medium has a sweep width W and each vehicle has a sweep velocity V. In addition we are given the density $f'(xyz)$ of targets in the operating space. The product of $W \cdot V \cdot f'(xyz)$ is the sweep rate S_L of the reconnaissance sub-system.

The sweep rate is an important characteristic of the reconnaissance sub-system. It is this characteristic which in a particular operating situation determines the reconnaissance ratio. This is done as follows:

(a) The sweep rate is determined as outlined in OEG 56* and appendix of this paper.

(b) The time spent sweeping per reconnaissance sub-system action, t_s,

* OEG 56, Chapter 2.

is determined from the characteristics of L and the given situation. Under certain conditions where the constraints on the reconnaissance sub-system do not determine its action, the time spent sweeping during each reconnaissance sub-system action may require optimization. We may assume, however, that in a given "overall weapons system" this time is a pre-determined system characteristic.

(c) The sweep time per reconnaissance sub-system action is multiplied by the sweep rate to find targets located per reconnaissance sub-system action.

$$\frac{\text{Targets}}{\text{reconnaissance action}} = t_s \cdot S_L = N_L$$

(d) From the characteristics of w and the given situation the average number of targets N_w, of class n which can be engaged per weapons sub-system action is determined. This quantity is limited by:

(1) The average number of targets within reach of w per action, adjusted for the time required for each engagement.

(2) The average number of engagements of which w is capable per action.

(3) A combination of 1 and 2. This number is determined for the "overall weapons system" when a given situation is specified.

Now, computing the ratio $\frac{N_w}{N_L}$ we have the reconnaissance ratio, N_{w_L}, the ratio of weapons sub-system actions to reconnaissance sub-system actions. This ratio is an optimum division of effort for the "overall weapons system" and, as was pointed out in Chapter II, the response of the decision mechanism will be actions of w and L in this ratio if

reconnaissance is warranted as determined by study of relative measures of effectiveness.

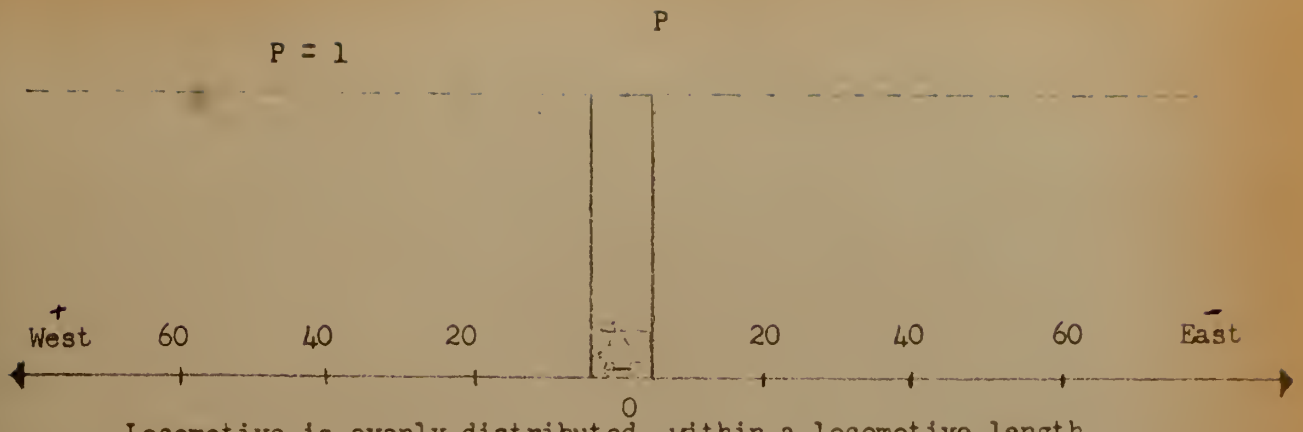
3. Target Information Deterioration

The quality of target information is highest at the time that the location and identity of a target in operating space is absorbed by a reconnaissance medium. We can say that at this time the target is located with certainty within the limits of reconnaissance medium resolving power.

If target n can have its position changed with time, the information absorbed by the reconnaissance medium will deteriorate with time. Consequently the probability P_c will also deteriorate with time, though this deterioration will not commence until the target has moved so that the uncertainty in its position is greater than the space in which P_c would be unity for w.

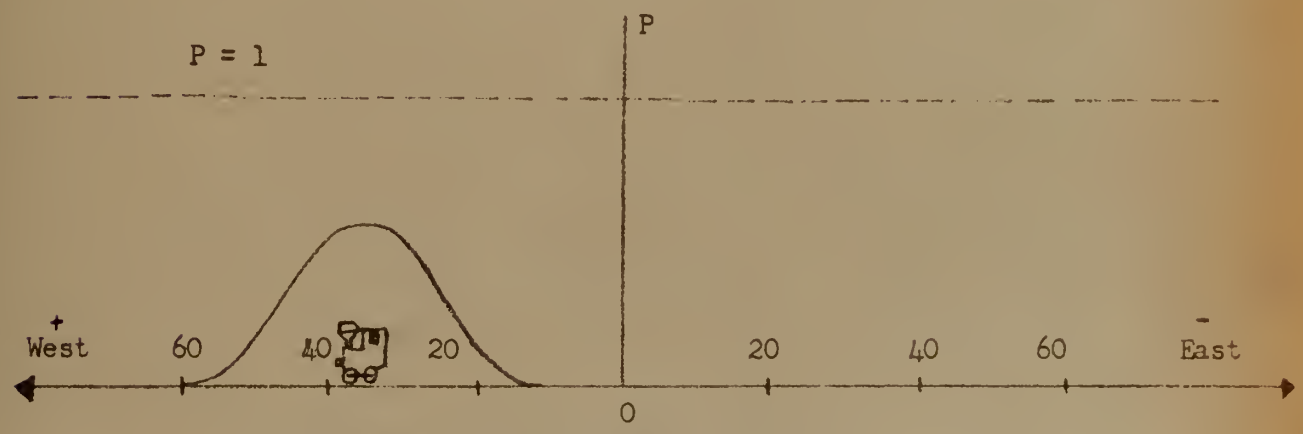
If target n cannot be moved, the quality of information, in the sense used here, does not deteriorate perceptibly during an operating period of the "overall weapons system".

Military targets may have zero, one, two or three degrees of freedom. For example, a train is restricted to motion along a definite line or in one dimension. A man on foot is restricted to movement in the plane of the earth's surface, or two dimensions. An aircraft may move in three dimensional space though its movement in the third dimension is so limited that a two dimensional model will suffice for most purposes. Wheeled and tracked vehicles move in ways that may have one or two degrees of freedom, depending upon the situation.



Locomotive is evenly distributed, within a locomotive length, (resolving power) around (0) at time located $P_{\underline{west}} = .8$

$P_{\underline{east}} = .2$ $\nu = 20$ m.p.h.



Locomotive is normally distributed about

$\mu =$ 36 miles, $\sigma = 27.7$ miles 3 hours after time located

Illustration 5

Example of one Demensional Deterioration
of
Target Location Information

Fixed installation targets such as buildings, emplacements and the like have zero degrees of freedom. Once located they will remain fixed as time passes.

In passing we may note that it is quite possible to think of qualitative changes in the target with time, the study of which should prove of value in considerations of strategic reconnaissance.

4. Deterioration of target information quality with time, in one dimension

In this paper the random walk model of diffusion as described by Feller* is used to approximate the motion of a moving target after detection. This approximation is best when the probabilities of motion forward and backward are nearly equal, when the velocity of the target can take on any value up to its maximum with equal probability and when the resulting probability distribution is used near its mean value.

Let us consider a one dimensional case. See illustration 5. A target moving with a velocity $v = \frac{\Delta X}{\Delta t}$ in a straight line and a probability p of moving in one direction and (1 - p) = q of moving in the other direction will, after a time t_d, be normally distributed about a mean, $\mu = t_d\,(p-q)\,\frac{\Delta X}{\Delta t} = 2ct_d$ with a variance $\sigma^2 = 4pq\left(\frac{\Delta X}{\Delta t}\right)^2 t = 2Dt_d$

The drift constant $c = \frac{1}{2}\,(p-q)\,\frac{\Delta X}{\Delta t}$ and the diffusion constant $D = 4pq\left(\frac{\Delta X}{\Delta t}\right)^2$, are target characteristics which describe the target distribution as a function of time.

5. Example of information deterioration

Suppose we are given the location, within a locomotive length, of a locomotive headed east on a very long east-west railroad track. We are told that the locomotive has a maximum velocity of 20 miles per hour

* Feller - Probability Theory and its Applications, Vol. I, p. 294-5.

and that the locomotive will move ahead with probability (.8) and backward with probability (.2). What is the probability distribution of the locomotive's position after three hours?

After three hours

μ = 3 hours (.8 - .2) 20 $\frac{\text{miles}}{\text{hour}}$ = 36 miles in direction of greatest probability.

σ^2 = 4(.8)(.2) · 3 hours x 400 $\frac{\text{miles}}{\text{hour}}$ = 768 (miles)

σ = $\sqrt{768}$ = 27.7 miles

The form of the distribution is shown in illustration 5.

6. Deterioration of target information with time in two dimensions

The bivariate normal distribution will serve as the model of target dispersion with time, in two dimensional space. This distribution may be calculated using one dimensional normal distributions along an x and y axis as marginal distributions of the desired bivariate distribution.

7. Deterioration time in the "overall weapons system".

Since time delays play such an important part in determining the deterioration of target information, we will now look at the "overall weapons system" to determine their source. Later, when optimizing the "overall weapons system" with respect to any of its components, the reduction of the time delays associated with the component will be of major importance. The delays by sub-system are:

(a) Reconnaissance sub-system (see illustration 6)

(1) t_s, average time spent sweeping after a target is found, or, time lost before the average piece of target information is put into the communications channel from the reconnaissance vehicle-medium to the information processing

component.

(2) t_r, average time spent by the reconnaissance vehicle-medium returning to the location of the information processing component, or, average time each piece of target information spends in the communications channel between the reconnaissance vehicle-medium and the information processing component.

(3) t_p, average time spent by each piece of target information in the information processing component.

(4) t_i, average time spent in the interpretation component by each piece of target information.

(5) t_q, average time spent by each piece of information in the communication channels between the information processing component and the decision mechanism.

(b) Decision Mechanism

(1) t_c, average time spent by decision mechanism deciding on action to be taken on each piece of target information received.

(2) t_o, average time spent phrasing and transmitting orders to the weapons sub-system.

(c) Weapons sub-system

(1) t_b, average time spent preparing to act.

(2) t_a, average time spent in action before the target is reached.

CHAPTER IV

OPTIMIZING THE "OVERALL WEAPONS SYSTEM WITH RESPECT TO ITS RECONNAISSANCE SUB-SYSTEM

1. The effect of variation of z, on the kill probability P_K.

In Chapter II we derived a measure of effectiveness for the "overall weapons system".

$$M.E. = M_{ax} \begin{cases} \dfrac{P_K}{C_{\underline{w}} + C_{\underline{L}}/N(\underline{\underline{w}}) + C_{\underline{d}} + C_{\underline{c}}} & 1 \text{ (using reconnaissance)} \\ \dfrac{P_K}{C_{\underline{w}} + C_{\underline{d}} + C_{\underline{c}}} & 2 \text{ (without reconnaissance)} \end{cases}$$

Let us assume that for our given situation equation 1 gives the maximum value. We now wish to optimize the "overall weapons system" with respect to its reconnaissance sub-system. This in effect allows us to vary C_L and $N(\underline{\underline{w}})$.

First we will see how P_K varies with changes in $P_K = P_{(KIC)} \cdot P_{(C)}$

P_c, the probability of a contact by w on n, is the only factor in the kill probability that is a function of the reconnaissance sub-system. Consequently it is the only factor in P_K we can vary in this optimization process.

Let us plot P_c vs. Z (see illustration 7(a). For Z = 0, P_c is a maximum, and as Z increases P_c decreases monotonically, approaching a constant asymptotically.

Since $P_{(KIC)}$ is a constant for a given $\underline{n}$ and $\underline{w}$; a plot of P_K vs. Z is proportional to that of P_c vs. Z. For $P_{KIC} = 1$ the two plots will be the same. For $P_{KIC} \leq 1$ the plot of P_K vs. Z will be everywhere less than P_c vs. Z. See illustration 7b.

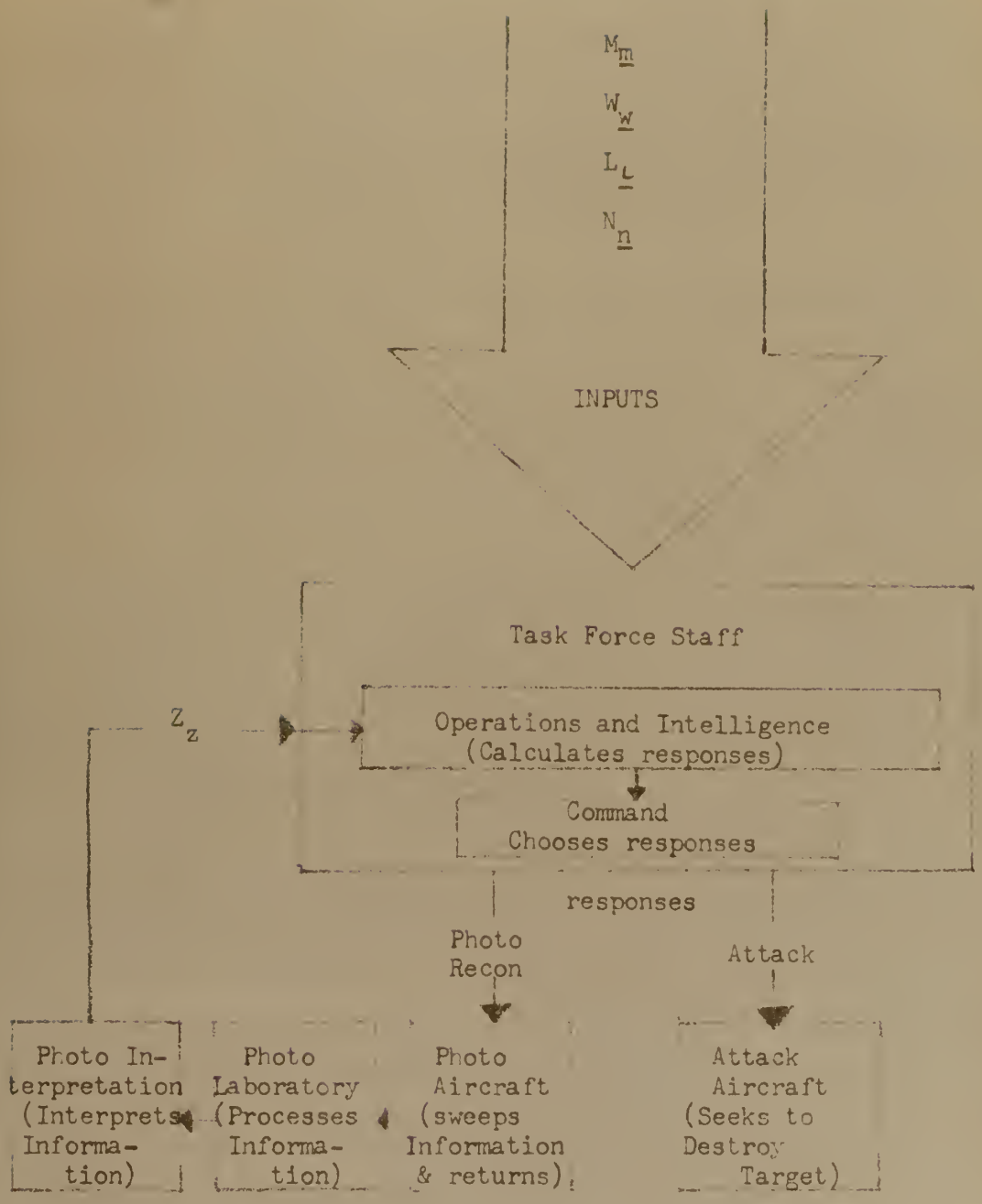


Illustration 6

The Fast Carrier Task Force as an

"Overall Weapons System"

In our model of the distribution of a moving target with time the deterioration, which we have previously denoted by Z, may be measured in terms of σ , the standard deviation. Consequently we may replace the absaissa Z by σ_n .

P_c itself is the product of two probabilities.

(a) P_f the probability that w will find n if it passes near it.

(b) P_s the probability that the searching operation by w will bring it over n.

So $P_c = P_f \cdot P_s$

P_s here is the integral over a portion of a normal distribution having $\sigma_n = 2Dt_d$ where t_d is the time delay in the "overall weapons system". Since the decision mechanism can calculate the mean position of n, we can credit w with searching equally on each side of the mean target position; then P_s at any time, t is $\frac{1}{2\pi\sigma_n}\int_{\mu-\frac{\Delta w}{2}}^{\mu+\frac{\Delta w}{2}} e^{-\frac{x^2}{2\sigma^2}} dx$ where Δw is the mean distance w can search for n.

P_f is independent of Z and therefore P_c and consequently P_K can be represented by a function proportional to P_s. A plot of P_K vs. Z will be proportional to a plot of P_c vs. t_d where each ordinate represents an integration of a different normal distribution.

2. The effect of variation of Z on the C_1, the cost of the reconnaissance sub-system

We have stated in the last section that Z is proportional to time delay. Therefore we are actually interested now in a function of cost of reconnaissance sub-system versus time delay. Or to restate the problem, what does it cost to decrease time delay when we are limited to changes in the reconnaissance sub-system only.

This function of cost versus t may be obtained empirically, as follows:

(a) Prepare a detailed listing of the time delays within the reconnaissance sub-system as was done in Chapter III.

(b) Place this list in one to one correspondence with the components of physical equipment or organizations making up the reconnaissance sub-system.

(c) Derive an expression for the time delay in each component of the reconnaissance sub-system in terms of its important physical characteristics such as speed, processing rate, etc.

(d) Devise methods of changing each of the characteristics, which affect individual time delays, in steps such that the time delay is reduced a certain percent. The percent reduction steps can be chosen to give desired accuracy.

(e) Calculate the cumulative cost for each time reduction step.

(f) Derive an expression for the sweep rate of the reconnaissance sub-system in terms of its physical characteristics.

(g) Devise a method of calculating sweep rate for each reconnaissance sub-system produced.

(h) Now arrange the material prepared in the above steps (a through g),,in a table with columns for sweep rate cost and deterioration time, (See illustration 8), versus % deterioration time decrease steps. Within the boxes in the table place,

(1) In the sweep rate column the values of sweep rate.

(2) In the deterioration time columns the original and improved (decreased) values of deterioration time and the cumulative

cost of decrease.

(i) Now make all possible combinations choosing one element from each column.

(j) For each combination calculate the cost, compute the total deterioration time and the sweep rate.

(k) Compute the reconnaissance ratio of each combination from sweep rate and system constants as in Chapter III, section 2d.

(l) Divide each combination cost by its own reconnaissance ratio.

(m) Now set up a time delay axis on which time class intervals are marked.

(n) Arrange the combination costs (adjusted for reconnaissance ratio), as ordinates in time classes corresponding to total deterioration.

(o) Now draw a step graph through the lowest ordinate in each time box. See illustration 9.

The function portrayed by the step graph is cost vs. time delay.

3. Determination of the optimal reconnaissance sub-system

Now having a function of P_K versus deterioration time and a function of cost versus deterioration time we can readily determine a function of "overall weapons system" effectiveness versus deterioration time. By dividing values of P_K by cost we have a reasonable measure of "overall weapons system" effectiveness which can be plotted as a function of deterioration time. See illustration 10.

This measure of effectiveness will have a maximum at some value of t_d. The reconnaissance sub-system corresponding to this value of t_d is the optimum; other parts of the "overall weapons system" remaining fixed.

CHAPTER V

ILLUSTRATION OF METHODS

1. General methods developed using the Fast Carrier Task Force as an example of an "overall weapons system".

The rigorous use of an optimization method such as the one developed in this paper requires extensive labor and the availability of considerable technical information as to cost, characteristics and feasibility of equipment and human organization. Therefore the preparation of an illustrative example even in a field with which the writer is quite familiar calls for the use of broa powers of simplification and estimation by the writer. In addition it has proved impossible to use much of the factual information available because of security classification of this paper.

2. The detailed composition of a Fast Carrier Task Force

A Fast Carrier Task Force when simplified and thought of as an "overall weapons system" can be divided into sub-systems and components as follows:

a. Reconnaissance sub-system - Photo reconnaissance system

(1) Reconnaissance vehicle medium - (Photo aircraft)

(2) Communications link between reconnaissance vehicle - medium and information processing component - (human runners).

(3) Information processing component - (Photographic laboratory)

(4) Communications link between information processing component and information interpreting component - (human runners).

(5) Information interpretation component - (human photographic interpreters).

(6) Communications link between information interpretation component and decision mechanism - (human runners).

b. Decision mechanism - (Task force staff, operation officer and Task force commander).

(1) Calculation of optimum and other responses from intelligence - (Operations department).

(2) Transmission of optimum responses to decisioner - (human runner - visual or aural message).

(3) Decisioner - choice of responses - (human).

(4) Transmission link for responses chosen by decisioner to the weapons sub-system - (human runner - written message - visual message - radio message).

c. Weapons sub-system - (Attack aircraft)

(1) Preparation of weapons sub-system for action - (briefing - arming, etc.).

(2) Proceeding to target - (flying time).

3. The "overall weapons system" inputs.

We will assume that we are given a Fast Carrier task force that is in a steady state combat operation. The inputs to the "overall weapoks system" are fixed. Suppose we have as in illustration 2:

a. Mission m, see illustration 2.

(1) Operate continuously in area A which is approximately 174 miles off the coast line of enemy.

(2) Destroy supply vehicles (n) moving along coastal supply route R from the boundary of a non-active power to a front line; a distance of 600 miles or more.

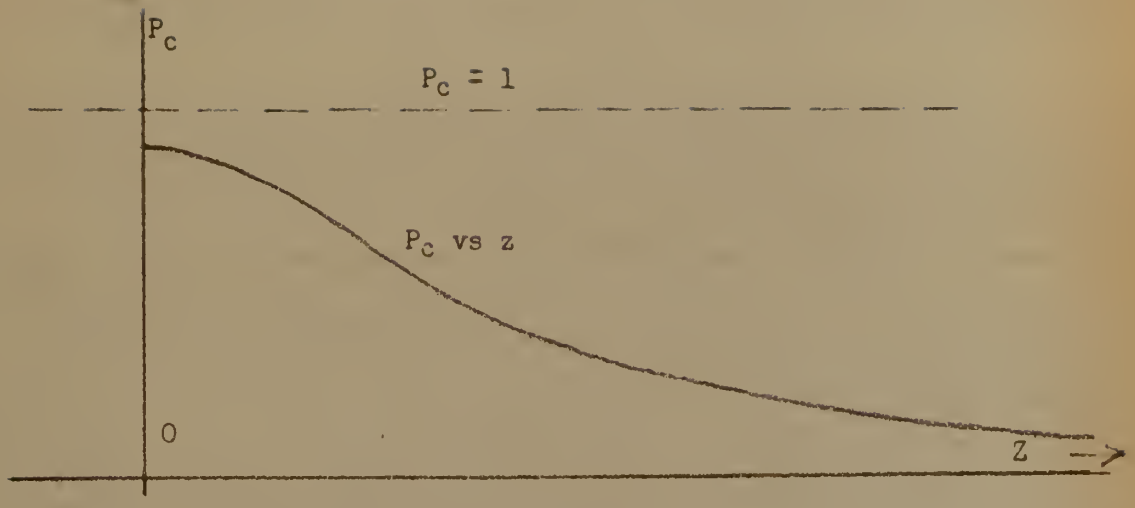


Illustration 7(a)

Probability of Contact P_c vs. Information Deterioration Z

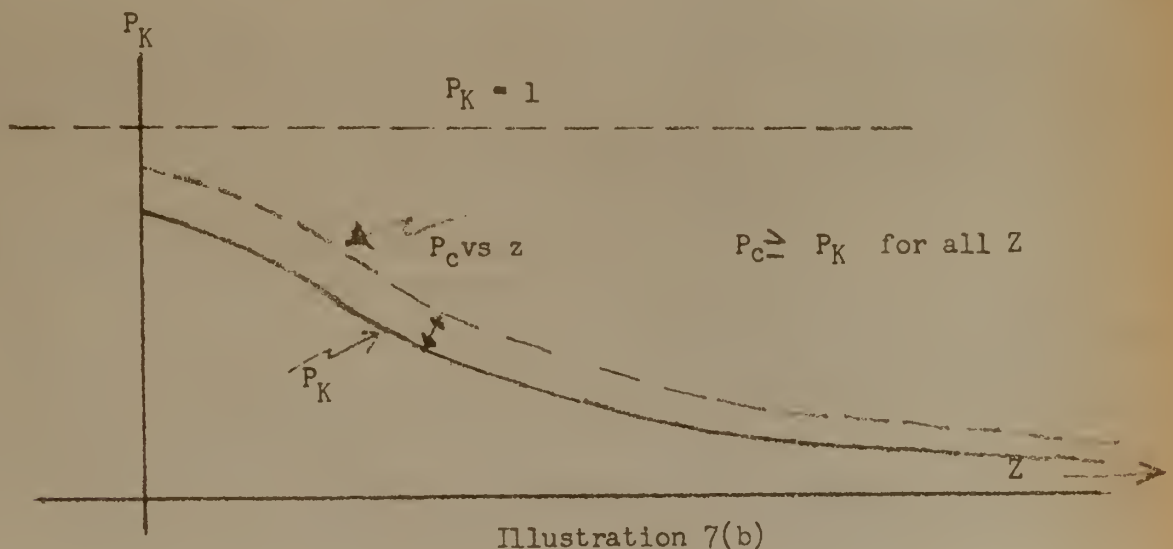


Illustration 7(b)

Probability of Kill P_K vs. Information Deterioration Z

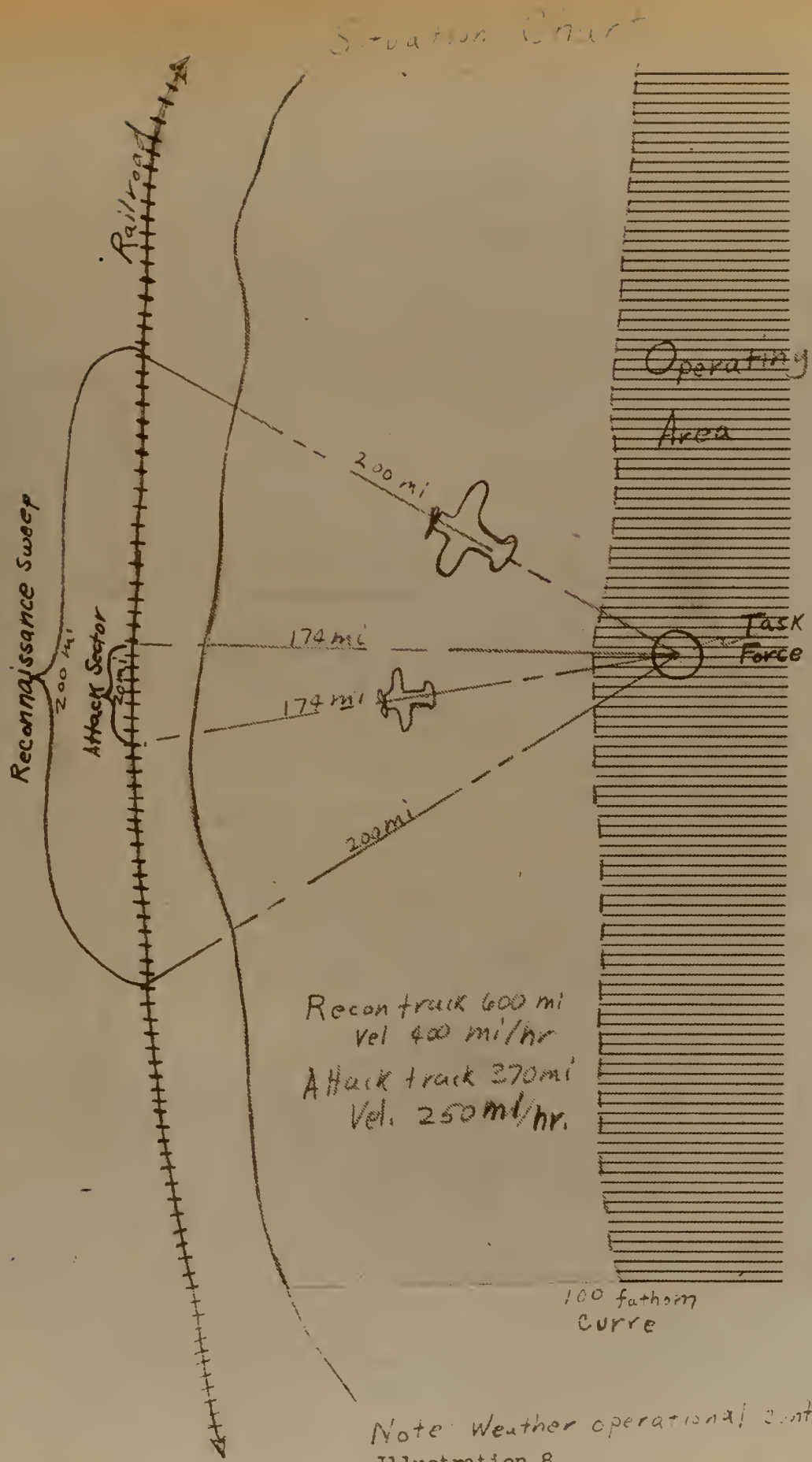


Illustration 8

(3) Maximize the kills of supply vehicles (n) while minimizing the dollar cost of operations.

b. Weapons sub-system characteristics w - attack aircraft

(1) Range - 200 miles

(2) Velocity - 250 miles per hour

(3) Search - Capable of searching a 20 mile segment of R with probability 1.of finding anything on the segment.k

(4) Kill - Capable of killing any target n found, with probability 1

(5) Briefing and readying time - 15 minutes.

(6) Cost of w per sortie, $2,000.

(7) Pro rata costs per mission of w $10,000.

c. Reconnaissance sub-system - (Photo reconnaissance) characteristics

(1) Reconnaissance vehicle-medium (Photo aircraft).

(a) Range-600 miles, 200 spent sweeping.

(b) Velocity-400 miles per hour

(c) Sweep rate - two objects per hour (we have assumed that the sweeping is perfect).

(d) Cost of Photo aircraft, $1,500 per mission.

(e) Pro rata costs per mission, $5,000.

(f) Altitude to sweep target n, with certainty is, 7000'.

(2) Communications link between vehicle-medium and information processing component - human runner requires 5 minutes to deliver film from aircraft on carrier flight deck to photographic laboratory.

(3) Information processing component - Shipboard photographic

laboratory overall processing rate .25 $\frac{\text{min}}{\text{ft}}$ -- time from raw negative to finished print. Pro rata costs in laboratory per reconnaissance mission, $500.

(4) Communications link between Information processing component and Information interpreting component - human runner requiring 5 minutes to deliver photographic prints from laboratory to interpretation office.

(5) Information interpretation component - Photographic interpreter, Interpretation rate $\frac{1 \text{ min}}{\text{ft}}$ - average time to locate target on photograph, calculate mean advance position of target, and delineate the search sector for one attack aircraft W. Pro rata cost of Interpretation, $100 per mission.

(6) Communications link between Information interpretation component and decision mechanism - human runners requiring 5 minutes to deliver interpretation reports from interpretation office to decisioner.

d. Time delays in decision mechanism.

(1) 10 minutes to calculate response and transmit to decisioner.

(2) 5 minutes to choose responses, phrase orders and transmit to weapons system.

e. Characteristics of target n.

(1) Max. velocity 20 miles per hour.

(2) Probability that n will continue to move in direction headed when detected .P = .8 in opposite direction q = .2

(3) Length of vehicle 20'

(4) Probability that n will be killed by w if found = 1.

(5) Density of targets n = .005 $\frac{\text{vehicles}}{\text{mile}}$

4. Calculation of total deterioration time of average operation.

From the situation and "overall weapons system" characteristics we will now calculate the deterioration time t_d.

a. t_s average time spent sweeping after a target is found.

$$t_s = \tfrac{1}{2} \frac{\text{mean length of photo sweep}}{\text{photo aircraft velocity}}$$

$$t_s = \tfrac{1}{2} \frac{200 \text{ miles}}{400 \text{ mile/hour}} = .25 \text{ hours}$$

b. t_r average time spent by photo aircraft returning from sweep.

$$t_r = \frac{\text{mean distance of photo aircraft from task force at end of sweep}}{\text{photo aircraft velocity}}$$

$$t_r = \frac{200 \text{ miles}}{400 \text{ miles/hour}} = .5 \text{ hours}$$

c. t_p average time spent processing each piece of target information.

Since film is processed in a complete roll all information must wait for complete process to be completed.

$$t_p = \frac{\text{processing time}}{\text{foot}} \cdot \frac{\text{mean length of photo sweep}}{\frac{\text{ground gained}}{\text{forward/exposure}} \cdot \frac{\text{miles}}{\text{photo foot}}}$$

$$t_p = (.25)\frac{\text{minutes}}{\text{foot}} \cdot \frac{200 \text{ miles}}{(.4) \cdot 1.3 \frac{\text{miles}}{\text{foot}}} = 96 \text{ min} = 1.6 \text{ hours}$$

d. t_i average time spent by each piece of target information in the interpretation component. Since each target is immediately reported to the decision mechanism the mean t_i is $\frac{1}{2}$ of the total time spent interpreting per sortie.

$$t_i = \tfrac{1}{2} \frac{\text{interpretation time}}{\text{foot}} \cdot \text{film length}$$

$$t_i = \tfrac{1}{2} \frac{1 \text{ minute}}{\text{foot}} \frac{200 \text{ miles}}{(.4) \cdot 1.3 \frac{\text{miles}}{\text{foot}}}$$

t_i = 192 min = 3.2 hours.

e. t_q average time spent by each piece of information in communication channels within the reconnaissance sub-system.

t_q = aircraft to lab , lab to interpreter , interpreter to decision mechanism

t_q = 5 min. + 5 min. + 5 min. = 15 min. = .25 hours

f. t_c average time spent computing optimum response

t_c = 10 minutes = .17 hours

g. t_o average time spent phrasing and transmitting orders to decision mechanism.

t_o = 5 minutes = .08 hours

h. t_b average time spent briefing and reading $\underline{w}$

t_b = 15 minutes = .25 hours

i. t_a average time spent flying to target

$$t_a = \frac{\text{average distance to search sector} + \frac{1}{2} \text{ sector length}}{(\text{attack aircraft velocity})}$$

$$t_a = \frac{174 + 10 \text{ miles}}{250 \frac{\text{miles}}{\text{hours}}} = .74 \text{ hours}$$

The total time deterioration is the sum of all the above factors.

t_d = 7.05 hours or $\approx$ 7 hours

5. Calculation of reconnaissance ratio

a. Sweep rate and targets per photo reconnaissance sortie.

The target density is given as .005 $\frac{\text{targets}}{\text{mile}}$. Since each reconnaissance sortie sweeps 200 miles of road we can calculate targets per reconnaissance sortie.

$$N_L = \frac{\text{targets swept}}{\text{mile}} \cdot \frac{\text{miles swept}}{\text{sortie}}$$

$$N_L \quad .005 \cdot 200 = 1 \quad \frac{\text{target found}}{\text{reconnaissance sortie}}$$

b. $N_{\underline{w}}$ is given as one target per attack aircraft sortie.

The reconnaissance ratio is then

$$N_L^W = \frac{N_W}{N_L} = 1$$

This means that one sortie of a photo plane is needed per sortie of an attack plane.

6. Calculation of target distribution.

We have been given that for n

p = .8

q = .2 v = 20 mph

We have calculated that t_d = 7 hours.

At the end of t_d the target n found in the reconnaissance sweep will be normally distributed with

μ = 36 miles

σ = 27.7 miles

as in the example of Chapter 3.

7. Calculation of P_K

We have been given that $P_{KIC} = 1$

Therefore since $P_K = P_c \cdot P_{KIC}$ $P_K = P_c$

We have also known that $P_c = 1$, if W happens to search where n is actually located. Since W can sweep only 20 miles of road and since the decision mechanism knows the mean position of target at all times we can expect W to search from ten miles ahead to ten miles behind the expected mean position of n.

Therefore:

In this case

with recon.

If we did not conduct reconnaissance, n would be equally likely to be located anywhere along the full 200 miles searched at any time. Therefore,

$$P_{C_W} = \frac{20 \text{ miles}}{200 \text{ miles}} = .1 = P_K$$

8. Calculation of costs.

a. C_W cost of attack aircraft per sortie.

C_W = dollar cost of attack aircraft per sortie.

prorated cost of training, upkeep, armament, etc. per sortie.

C_W = \$2000 + \$10,000 = \$12,000

b. C_L cost of reconnaissance sub-system per sortie.

C_L = dollar cost of reconnaissance aircraft per sortie.

pro rated cost of training, upkeep, etc. per reconnaissance aircraft per sortie

pro rated cost of processing per sortie

pro rated cost of interpreting per sortie

C_L = \$1500 + \$5000 + \$500 + \$100

C_L = \$7100

c. C_d cost of decision

C_d = \$100

d. C_c cost of communications

C_c = \$100

9. Calculation of measure of effectiveness and "overall weapons system" responses.

$$ME = \text{Max} \quad \frac{P_K}{C_W + C_{L/N_W} + C_d + C_c} \quad \text{with reconnaissance}$$

$$\frac{P_K}{C_W + C_d + C_c} \quad \text{without reconnaissance}$$

$$ME = \text{Max}\begin{cases} \dfrac{.2}{\$19,300} = 1.03 \times 10^{-5} \ \dfrac{\text{kills}}{\$} & \text{reconnaissance} \\ \dfrac{.1}{\$12,200} = .82 \times 10^{-5} \ \dfrac{\text{kills}}{\$} & \text{no reconnaissance} \end{cases}$$

The decision mechanism will choose to send out one photo reconnaissance sortie per attack sortie under these conditions. Letting P_K vary we can see that when $P_c(Lw) \geq .158$ decisioner will no longer reconnoiter for targets n since the attack aircraft can do as good a job by itself.

Letting C_L vary, decisioner will no longer reconnoiter for targets n when total cost $\geq$ \$24,400.

Suppose that N_{w_L} were two instead of one indicating a greater target density, then:

$$ME = \text{Max}\begin{cases} \dfrac{.2}{15,750} = 1.27 \times 10^{-5} \ \dfrac{\text{kills}}{\$} & \text{reconnaissance} \\ \dfrac{.2}{12,200} = 1.64 \times 10^{-5} \ \dfrac{\text{kills}}{\$} & \text{no reconnaissance} \end{cases}$$

Now the decisioner will decide to send attack aircraft to search the route without prior reconnaissance.

Suppose that with N_{w_L} still equal to 2 we were to cut the "overall weapons system" time delays from seven to four hours.

Then $P_K = 2.8$

$$ME = \text{Max}\begin{cases} \dfrac{2.8}{15,750} = 1.78 \times 10^{-5} \ \dfrac{\text{kills}}{\$} & \text{reconnaissance} \\ \dfrac{.2}{12,200} = 1.64 \times 10^{-5} \ \dfrac{\text{kills}}{\$} & \text{no reconnaissance} \end{cases}$$

Here again it is slightly profitable to carry on reconnaissance.

We may conclude then that it is only marginally profitable to conduct reconnaissance for targets of type n under conditions of this problem. If the target density increases reconnaissance becomes unprofitable. If

the "overall weapons system" time delays are reduced reconnaissance becomes more profitable.

10. Optimizing the photo-reconnaissance attack aircraft "overall weapons system"

a. Calculation of the function P_K versus time.

We have shown earlier in this chapter that under the assumptions made in this example $P_K = P_c = \frac{1}{\sqrt{2\pi}\sigma_n} \int_{-10}^{+10} e^{-\frac{x^2}{2\sigma_n^2}} dx$

b. Calculating values of σ_n for increasing values of t_d and making the above integration we can plot a graph of P_K versus t. See illustration 10.

b. Calculation of the function C_L versus time.

Using the technique described in Chapter IV plot the step function C_L versus t. See illustration 9.

c. Determination of optimum t_d from M.E. versus t_d

Divide the function P_K versus t by the function of C total cost versus t. The resulting function is M.E. versus t and its maximum will indicate the optimum t_d for the "overall weapons system". See illustration 11.

d. Determination of optimum reconnaissance sub-system from t_d and C_L versus t_d.

The optimum reconnaissance sub-system is that prescribed by the component combination described by the value of C_L versus t_d at the value of t_d giving maximum effectiveness. In our example the optimum value of t_d is .96 hours and the optimum component combination leading to this t_d is (anoh) from Table 8b.

Table for Computation of Sub-System

Component Improvement Costs

Hypothetical Fast Carrier Task Force

Reconnaissance Sub-System

Component and associated time delay cost & sweep rate if relevant	Photographic Reconnaissance Aircraft (ts tr)			Photographic Laboratory t_p		Photographic Interpretation t_i		All Communication t_q	
Sub-System with no improvement (0)%	Description (a)			(b)		(c)		(d)	
	$t_s + t_r$	$/sortie	S	t_p	$/sortie	t_i	$/sortie	t_q	$/sortie
	.75 hr	$1500	1	1.6 hr	$500	3.2 hr	$100	.25	$50
Cheapest 33% improvement		(e)		(f)		(g)		(h)	
	$t_s + t_r$	$/sortie	S	t_p	$/sortie	t_i	$/sortie	t_q	$/sortie
	.50 hr.	$7,500	1	1.07 hr.	$1000	2.13 hr.	$500	.16	$100
Cheapest 66% improvement		(i)		(j)		(k)		(l)	
	$t_s + t_r$	$/sortie	S	t_p	$/sortie	t_i	$/sortie	t_q	$/sortie
	.25 hr	$75,000	1	.53	$3000	1.07	$2000	.08	$500
Cheapest 99% improvement	$t_s + t_r$	(m) $	S	Processing during flight t_p	(n) $/sortie	t_i	(o) $/sortie	t_q	(p) $
					$6000	.05	$5000		

Illustration 9a

Table of combinations with associated time delay
Cost and Reconnaissance Ratio

No	Comb	td	$C	NW	No	Comb	td	Cl	NW	No	Comb	td	C	NW
1	abcd	6.8	2150	1	49	ebcd	5.55	8150		97	ibcd	5.30	78,650	
2	abch	5 71	2200		50	ebch	5.46	8200		98	ibch	5.21	75,700	
3	abcl	5.63	2600		51	ebcl	5.38	8500		99	ibcl	5.13	76,100	
4	abgd	4.73	2550		52	ebgd	4.48	8550		100	ibgd	4.23	76,C50	
5	abgh	4.64	2600		53	ebgh	4.39	8600		101	ibgh	Always greater		
6	abgl	4.56	3000		54	ebgl	4.31	9000		102	ibgl	than other		
7	abkd	3.67	4050		55	ebvd	3.42	10050		103	ibkd	values		
8	abkh	3.58	4100		56	ebkh	3.33	10100		104	ibkh	already cal-		
9	abkl	3.5	4500		57	ebkl	3.25	10500		105	ibkl	culated		
10	abod	2.65	7050		58	ebod	2.40	13050		106	ibod			
11	aboh	2.56	7100		59	eboh	2.31	13100		107	iboh			
12	abol	2.48	7500		60	ebol	2.23	13500		108	ibol			
13	afcd	5.27	2650		61	efcd	5.02	8650		109	ifcd			
14	afch	5.18	2700		62	efch	4.93	8700		110	ifch			
15	afcl	5.1	3100		63	efcl	4.85	9100		111	ifcl			
16	afgd	4.2	3050		64	efgd	3.95	9050		112	ifgd			
17	afgh	4.11	3100		65	efgh	3.86	9100		113	ifgh			
18	afgl	4.04	3500		66	efgl	3.78	9500		114	ifgl			
19	afkd	3.14	4550		67	efkd	2.89	10550		115	ifkd			
20	afkh	3.05	4600		68	efkh	2.80	10600		116	ifkh			
21	afkl	2.97	5000		69	efkl	2.72	11000		117	ifkl			
22	afod	2.12	7550		70	efod	1.87	13550		118	ifod			
23	afoh	2.03	7600		71	efoh	1.78	13600		119	ifoh			
24	afol	1.95	8000		72	efol	1.70	14000		120	ifol			
25	ajcd	4.73	4650		73	ejcd	3.48	10650		121	ifcd			
26	ajch	4.64	4700		74	ejch	3.39	10700		122	ijch			
27	ajcl	4.56	5100		75	ejcl	3.32	11100		123	ijcl			
28	ajgd	3.66	5050		76	ejgd	3.41	11050		124	ijgd			
29	ajgh	3.57	5100		77	ejgh	3.32	11100		125	ijgh			
30	ajgl	3.49	5500		78	ejgl	3.24	11500		126	ijgl			
31	ajkd	2.6	6550		79	ejkd	2.35	12550		127	ijkd			
32	ajkh	2.51	6700		80	ejkh	2.26	12600		128	ijkh			
33	ajkl	2.43	7000		81	ejkl	2.18	13000		129	ijkl			
34	ajod	1.58	9550		82	ejod	1.33	15550		130	ijod			
35	ajoh	1.49	9600		83	ejoh	1.24	15600		131	ijoh			
36	ajol	1.41	10000		84	ejol	1.16	16000		132	ijol			
37	ancd	4.2	7650		85	encd	3.95	13650		133	incd			
38	anch	4.11	7700		86	ench	3.86	13700		134	inch			
39	ancl	0.03	8100		87	encl	3.78	14100		135	incl			
40	angd	3.13	8050		88	engd	2.88	14050		136	ingd			
41	angh	3.04	8100		89	engh	2.79	14100		137	ingh			
42	angl	2.96	8500		90	engl	2.71	14500		138	ingl			
43	ankd	2.07	9550		91	enkd	1.82	15550		139	inkd			
44	ankh	1.98	9600		92	enkh	1.73	15600		140	inkh			
45	ankl	1.90	10000		93	enkl	1.65	16000		141	inkl			
46	anod	1.05	12550		94	enod	.80	18550		142	inod	.55	80050	
47	anoh	.96	12600		95	enoh	.71	18600		143	inoh	.46	80100	
48	anol	.88	13000		96	enol	.63	19000		144	inol	.38	80500	

Illustration 9b

Total Costs vs. Deterioration Time

Interval	Comb.	t_d	$\$C_L/N_L^W$
0 - .2			
.2 -.4	144	.38	80,500
.4 -.6	142	.55	80,050
.6 -.8	94	.80	18,550
.8 -1.0	47	.96	12,600
1.0-1.2	46	1.05	12,550
1.4-1.6	34	1.58	9,550
1.8-2.0	24	1.95	8,000
2.0-2.2	22	2.12	7,550
2.2-2.6	32	2.51	6,700
2.6-2.8	31	2.6	6,550
2.8-3.0	21	2.97	5,000
3.0-3.2	19	3.14	4,550
3.2-3.6	8	3.58	4,100
3.6-3.0	7	3.67	4,050
3.8-4.2	16	4.2	3,050
4.2-4.6	6	4.56	3,000
4.6-4.8	4	4.73	2,550
4.8-5.8	2	5.71	2,200
5.8-6.8	1	6.8	2,150

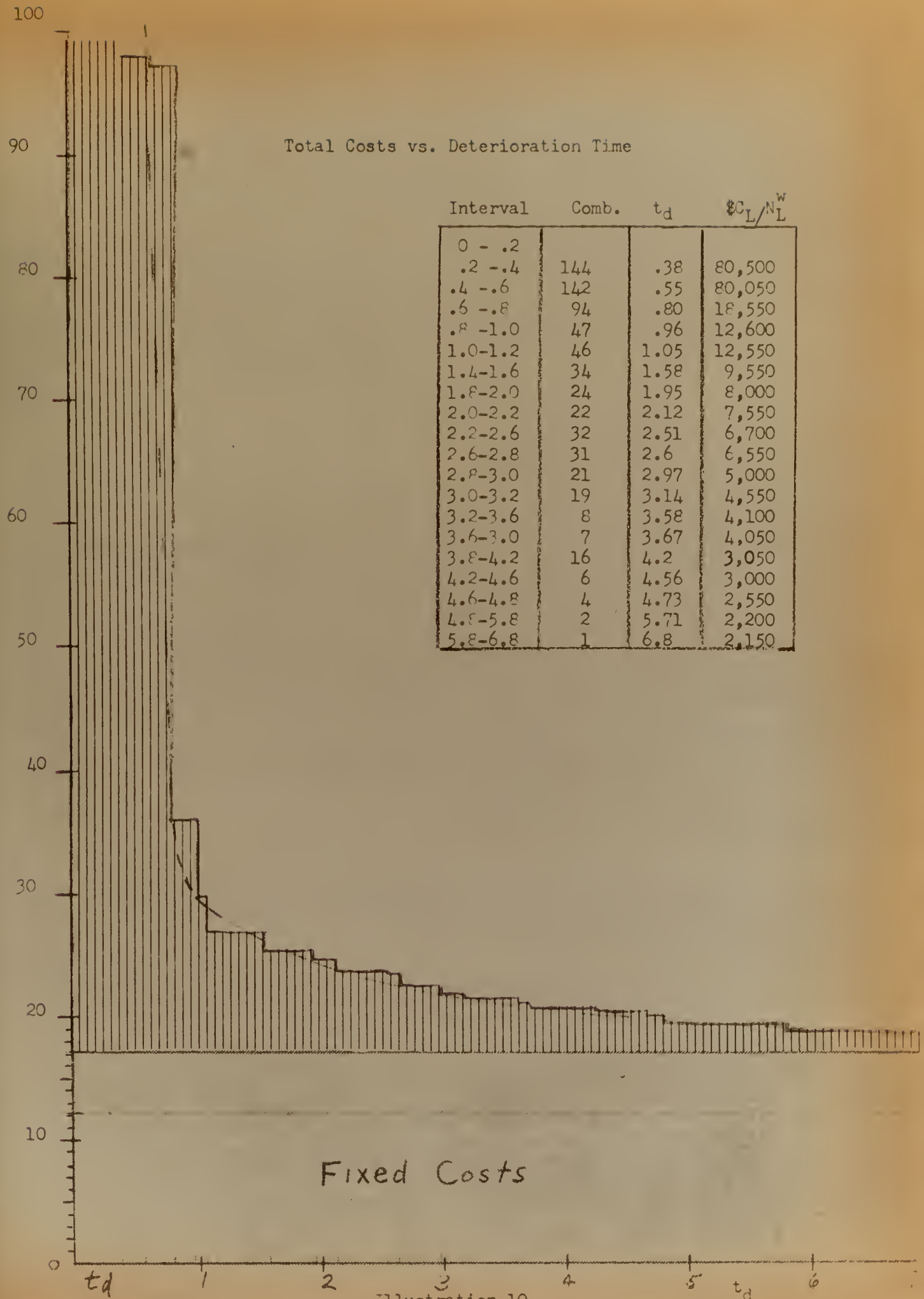


Illustration 10

Kill probability vs t_d

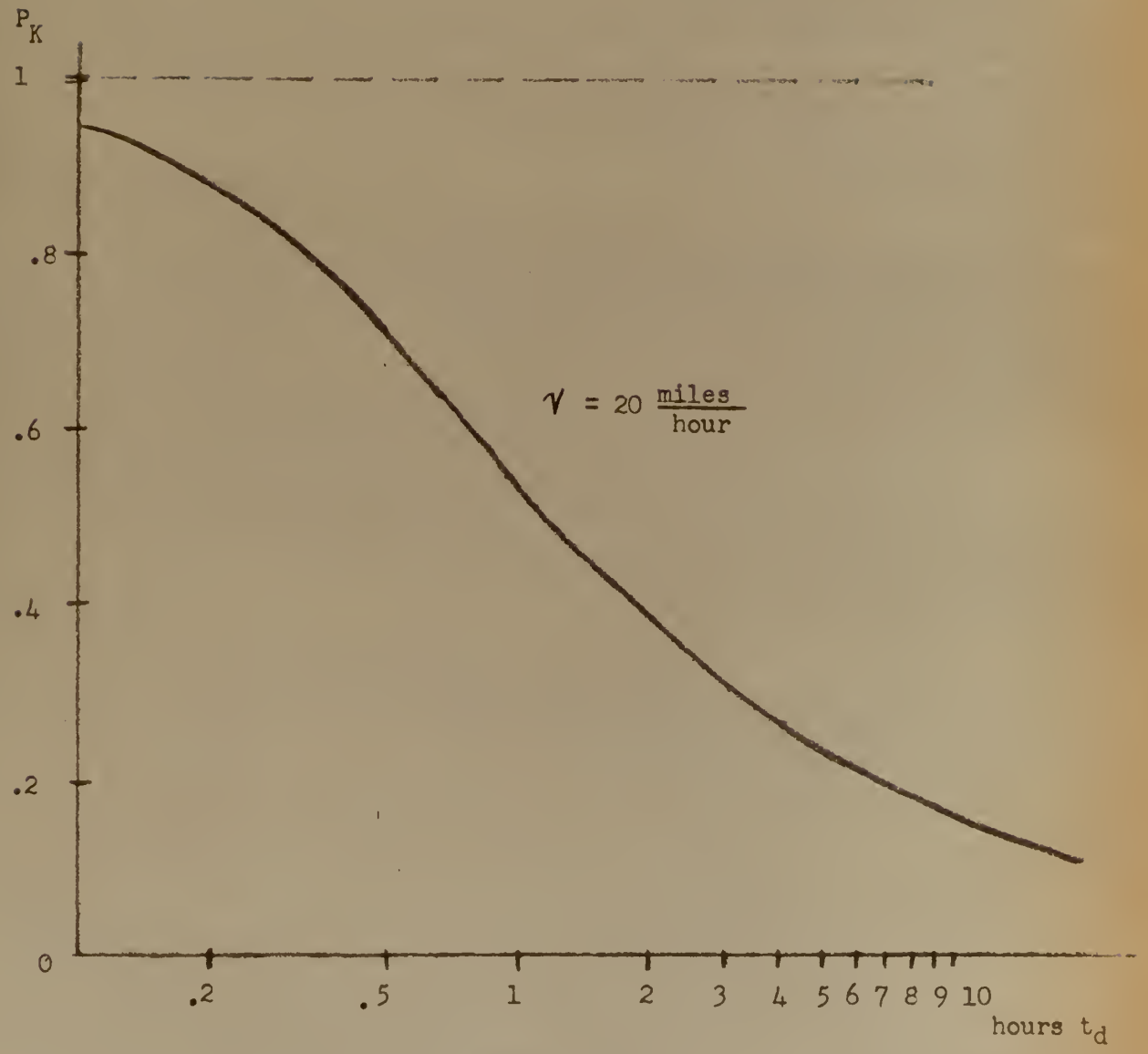


Illustration 11

t_d	C/N^w	$\frac{C_{fixed}}{N}$ costs/so	$K(n/wZ)ME \times 10^{-5}$		Comb.	No.
.38	80,500	97,650	.76	0.78×10^{-5}		144
.55	80,050	97,200	.68	0.7×10^{-5}		142
.80	18,550	35,700	.58	1.63×10^{-5}		96
.96	12,600	29,750	.53	1.79×10^{-5}	anoh	47
1.05	12,550	29,700	.51	1.73×10^{-5}		46
1.58	9,550	26,700	.43	1.61×10^{-5}		34
1.95	8,000	25,150	.40	1.59×10^{-5}		24
2.12	7,550	24,700	.38	1.535×10^{-5}		22
2.51	6,700	23,850	.35	1.465×10^{-5}		32
2.6	6,550	23,700	.34	1.43×10^{-5}		31
2.97	5,000	22,150	.32	1.445×10^{-5}		21
3.14	4,550	21,700	.31	1.43×10^{-5}		19
3.58	4,100	21,250	.295	1.39×10^{-5}		8
3.67	4,050	21,200	.290	1.36×10^{-5}		7
4.2	3,050	20,200	.270	1.34×10^{-5}		16
4.56	3,000	20,150	.260	1.28×10^{-5}		6
4.73	2,550	19,700	.250	1.27×10^{-5}		4
5.71	2,200	19,350	.230	1.14×10^{-5}		2
6.80	2,150	19,300	.20	1.03×10^{-5}		1

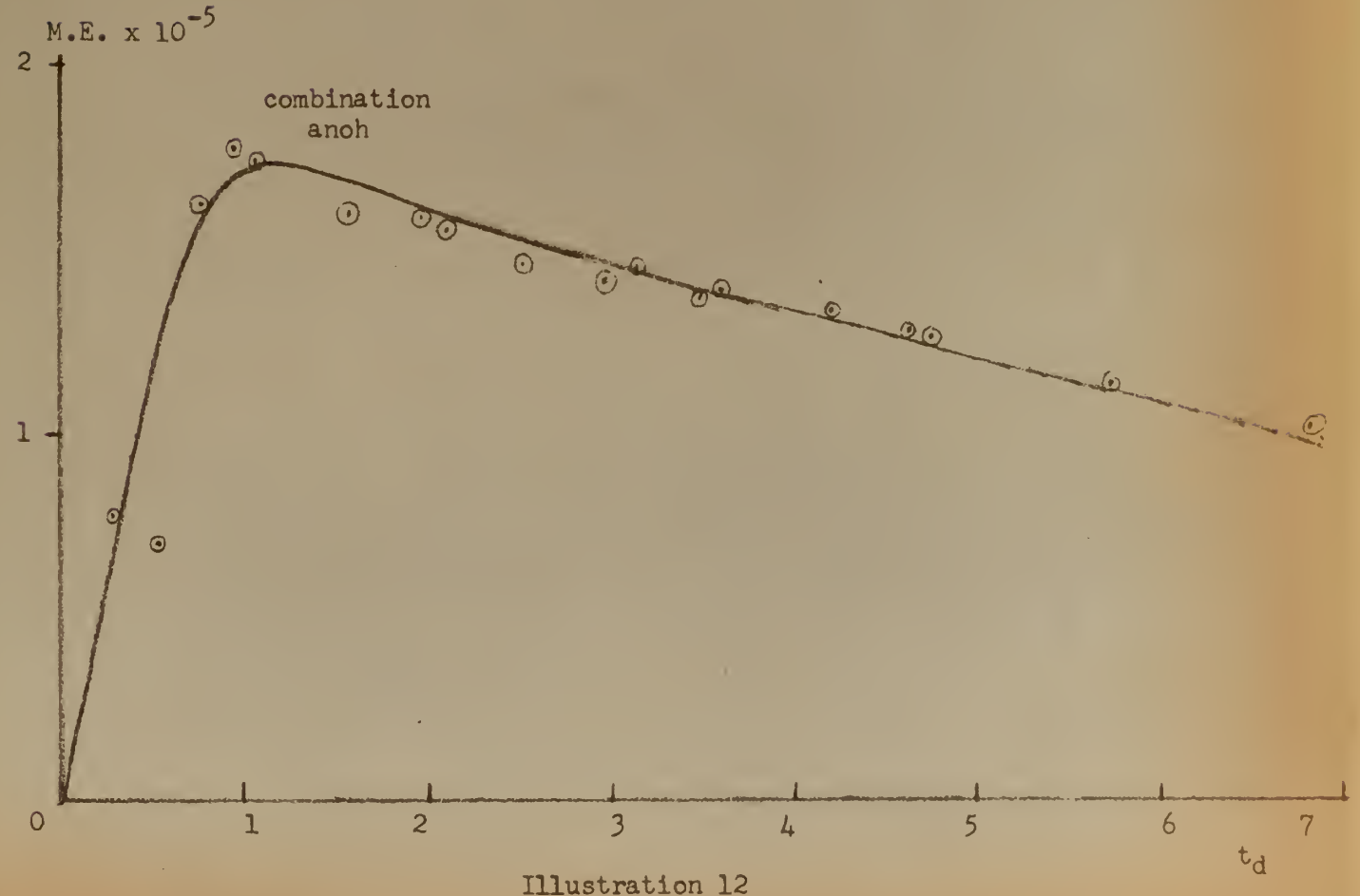


Illustration 12

CHAPTER VI

CONCLUSIONS

1. Tactical uses of "overall weapons system" model

For various real operational situations this model could be used by a commander to determine which of a number of target classes (n_1, n_2 . . n_K) of relative values (V_1, V_2 . . V_K) specified in a mission affords the best payoff for each of the various "overall weapons systems" formable from sub-systems available.

We can also determine the conditions on each of (n_1, n_2 . . n_m) for which a given "overall weapons systems" telescopes, i.e., reverts to no reconnaissance. This occurs when the measure of effectiveness with reconnaissance is less than that without reconnaissance.

2. Strategic uses of "overall weapons system" model

The model may be used for calculation of optimal "overall weapons system" characteristics with respect to

a. Reconnaissance sub system (L_1, L_2 . .L_j)

b. Weapons sub system (w_1, w_2 . . w)

c. Target class (n_1 . . . n_K)

d. Mission (m_1, m_2 . . m_n)

e. Any combination of the above.

Such calculations may lead to:

(1) Development of new sub-system or sub-system components.

(2) Re-combination of existing sub-systems and reassignment of targets and missions.

(3) Redistribution of forces to place "overall weapons systems" in

areas where targets for which they are optimal are most abundant.

(4) Restatement of missions so that national objectives may be best served by existing or feasible "overall weapons system".

BIBLIOGRAPHY

1. Boston University Optical Research Laboratory--Technical Notes 85 and 109.

2. Feller - Probability Theory and its Applications.

3. Greenleaf - Photographic Optics.

4. Koopman - Search and Screening. Chapter 2 (unclassified).

APPENDIX

SWEEP RATE

1. General discussion of sweep rate.

Every reconnaissance vehicle-medium will have an information sweep rate in targets/unit time. This rate S_L depends on:

a. The length of path covered by the vehicle per unit time, V_L.

b. The sweep width of the reconnaissance medium, W* which is a function of the following:

(1) The size, usable area, strength reflectance, or other target characteristic chosen to fit the reconnaissance medium.

(2) The capacity of the reconnaissance medium.

(3) The ratio of signal level to background level of the target, i.e., contrast, signal to noise ratio, etc.

c. The distribution of the target n versus size and signal to background ratio.

2. Air reconnaissance mediums.

From the morphologic Diagram of Airborne Reconnaissance (see illustration 13) we have the following reconnaissance media:

a. Visual search.

b. Day photography.

c. Night photography.

d. Cartographic photography.

e. Search radar.

f. Radar detectors.

g. Mapping radar.

* OEG 56. Chapter II.

MORPHOLOGIC DIAGRAM OF AIRBORNE RECONNAISSANCE

MILITARY SITUATION	A	B	C
NAVAL MISSION	STRATEGIC	TACTICAL	DEFENSIVE
	DESTROY NAVAL STRENGTH SOURCES OF ENEMY (OPERATING BASES; SHIP YARDS AND DOCKS; SUPPLY DEPOTS; AIRFIELDS; SEADROMES)	DESTROY NAVAL POWER (ATTACK SHIPS SUBS AIRCRAFT)	PREVENT ENEMY AIR ATTACK ON US AND ALLIES (DESTROY RAIDS)
	DESTROY SHIPPING CAPABILITY (PORTS AND HARBORS)	DESTROY MERCHANT SHIPPING (ATTACK SHIPS AT SEA)	ASSIST IN PREVENTION OF AIR ATTACK (PROVIDE A.E.W.)
	RESTRICT SEABORNE FORCES & TRADE (OBSTRUCT CANALS; BLOCKADE PORTS)	SEIZE BEACHHEADS (ATTACK BEACHHEAD OBJECTIVES AND NEUTRALIZE DEFENSES)	DEFEND BEACH HEADS (PROVIDE AND EMPLOY DEFENDING FORCES AT BEACHHEAD)
	PREVENT CONCENTRATION OF ENEMY (ATTACK ON A WIDE FRONT)	SEIZE ADVANCE NAVAL BASES (ATTACK SPECIFIC OBJECTIVES)	DEFEND ADVANCE NAVAL BASES
BASE LOCATION	0 200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 MAX RANGE		
BASE TYPE	AIRBORNE BASE	CARRIER DECK	LAND FIELD / SEA SURFACE
ATTACK WEAPON	SPECIAL BOMBS	EXPLOSIVES	INCENDIARIES	MINES	TORPEDOES	UNGUIDED MISSILES	GUIDED MISSILES	GUNS	RADIOLOGICAL	BACTERIOLOGICAL
ENEMY COUNTER MEASURES	FIGHTER INTERCEPT	ANTI-AIRCRAFT GUNS OR ROCKETS	ELECTRONIC COUNTERMEASURES	CONCEALMENT	GUIDED MISSILES	RADIOLOGICAL	DISPERSAL	PROPAGANDA
DEFENSIVE ARMAMENT	GUNS	ARMOR	SELF SEALING TANKS	GUIDED MISSILES	UNGUIDED MISSILES	RADIOLOGICAL	FIRE SMOTHERING SYSTEMS	SPEED
RECON. MEDIUM	VISUAL	DAY PHOTO	NIGHT PHOTO	CARTOGRAPHIC PHOTO	SEARCH RADAR	RADAR DETECTOR	RADAR MAPPING	COMMUNICATIONS MONITORING	COUNTERMEASURE DETECTION & MONIT.	MAD	PATHFINDING BEACON PLANTING	WEATHER INSTRUMENTS	SHADOWING & TRACKING	EARLY WARNING	EARLY EARLY WARNING
MISSION ALTITUDE	10,000 20,000 30,000 40,000 50,000 60,000 70,000 80,000 90,000 100,000														
AMOUNT OF PRIOR INFO.	25% 50% 75% 100%														
TIME LAG TILL USE	1 HOUR; 12 HOURS; 1 DAY; 2 DAYS; 3 DAYS; 5 DAYS; 10 DAYS; 15 DAYS; 20 DAYS; 30 DAYS; 60 DAYS; 90 DAYS; 6 MONTHS; 1 YEAR; 2 YEARS; 3 YEARS														

III ustration 13
47

h. Communications monitoring gear.

i. Counter measure detection and monitoring.

j. Magnetic Airborne detection and airborne scintillation detection.

*k. Pathfinding and beacon planting.

l. Weather instruments.

*m. Shadowing and tracking.

*n. Early warning.

*o. Early early warning.

All of these reconnaissance media except those marked * have sweep widths. These specialized (*) media of airborne reconnaissance must be evaluated by other means. Sweep width for visual radar and electronic mediums has been investigated to a considerable degree and extensive literature is available for the calculation of sweep rates of "overall weapons systems" utilizing these mediums. Since we are using the photographic reconnaissance example it is desirable to develop sweep rate using this medium.

3. Sweep rate of reconnaissance sub-system using photographic aircraft.

a. R_{max}, Maximum airdistance to target.

The Optical Research Laboratory of Boston University ** has shown that the differential threshold of target detectibility on a photograph $\frac{\Delta I_f}{I_f}$ can be expressed as follows:

$$\frac{\Delta I_f}{I_f} = C\, \rho^{K} (U A)$$

where I_f = intensity of target surrounding on film

ΔI_f = difference in intensity between target and target surroundings on film

C and K = constants of the reconnaissance sub-system.

** Boston University Technical Notes 85, 109.

P = Perimeter of the target.

UA = Useful area of the target defined as the area within 1.5 minutes from target edge.

If we have a fairly regular target, which is usually the case, the useful area is related to P_f, the target perimeter on the film, in a simple way. $UA \approx P_f \cdot f \cdot 43.63 \times 10^{-5}$ cm^2

P_f, the perimeter on the film is related to P_a, the perimeter of the object on the ground as follows:

$$P_f = P_a \frac{f}{R_{max}}$$

where f = focal length of camera

R_{max} = maximum air distance to target.

$\frac{\Delta I_f}{I_f}$, the differential threshold of target detectability on film is related to $\frac{\Delta I_a}{I_a}$, the actual differential threshold of target detectability on the ground as follows:

$$\frac{\Delta I_f}{I_f} = \frac{\Delta I_a}{I_a} e^{\beta R_{Max}}$$

where β is the attenuation coefficient and R_{max} the air distance to the target.

Using the above relations we can now find a transcendental equation for R_{max} to achieve detection and recognition of a target on film when we know actual size and shape and the minimum value of $\frac{\Delta I_a}{I_a}$ for the target and its background. We must also know the constants C and K.

$$\frac{\Delta I_a}{I_a} e^{\beta R} = C \left(P_a \frac{f}{R}\right)^K \cdot P_a \frac{f}{R} \cdot f \cdot 43.63 \times 10^{-5}$$

For a given situation then, a maximum value of R_{max} will satisfy this equation. For this R_{max} and any smaller value of R, all targets with the

characteristics given above will be detected by the reconnaissance sub-system. Here we think of the reconnaissance sub-system as including photo aircraft, processing and interpretation facilities, as in Illustration 14.

b. Day photo sweep width.

Let us consider that we have a photographic aircraft with an array of cameras which allow adjustment of coverage angle. (see illustration 15).

h = flying altitude

R_{max} = air distance to extreme of coverage

α = dip angle of coverage

γ = coverage angle

During daylight, illumination of the target and its surroundings is not limiting. Modern cameras with good image motion compensation devices allow exposure times sufficient to insure optimum exposure time. Automatic exposure control devices will, in the future, insure that the optimum exposure time is known.

To determine sweep width the value of R_{max} is calculated as in part (a).

Then $h = R_{max} \sin \alpha$ and $W = 2 R_{max} \cos \alpha$.

c. Day photo sweep rate.

S_{pr}, sweep rate for photo reconnaissance is the product of W_{dp}, sweep width for day photography V, the aircraft velocity, $\rho_{\underline{n}}(x,yz)$ the target density, and t_s, the time of sweep.

$$S_{pr} = W_{dp} \cdot V \cdot \rho_n(x,y,z) \cdot t_s = \frac{\text{targets}}{\text{time}}$$

d. Night photographic reconnaissance sweep width.

The value of R_{max} from which night photographic sweep width is to

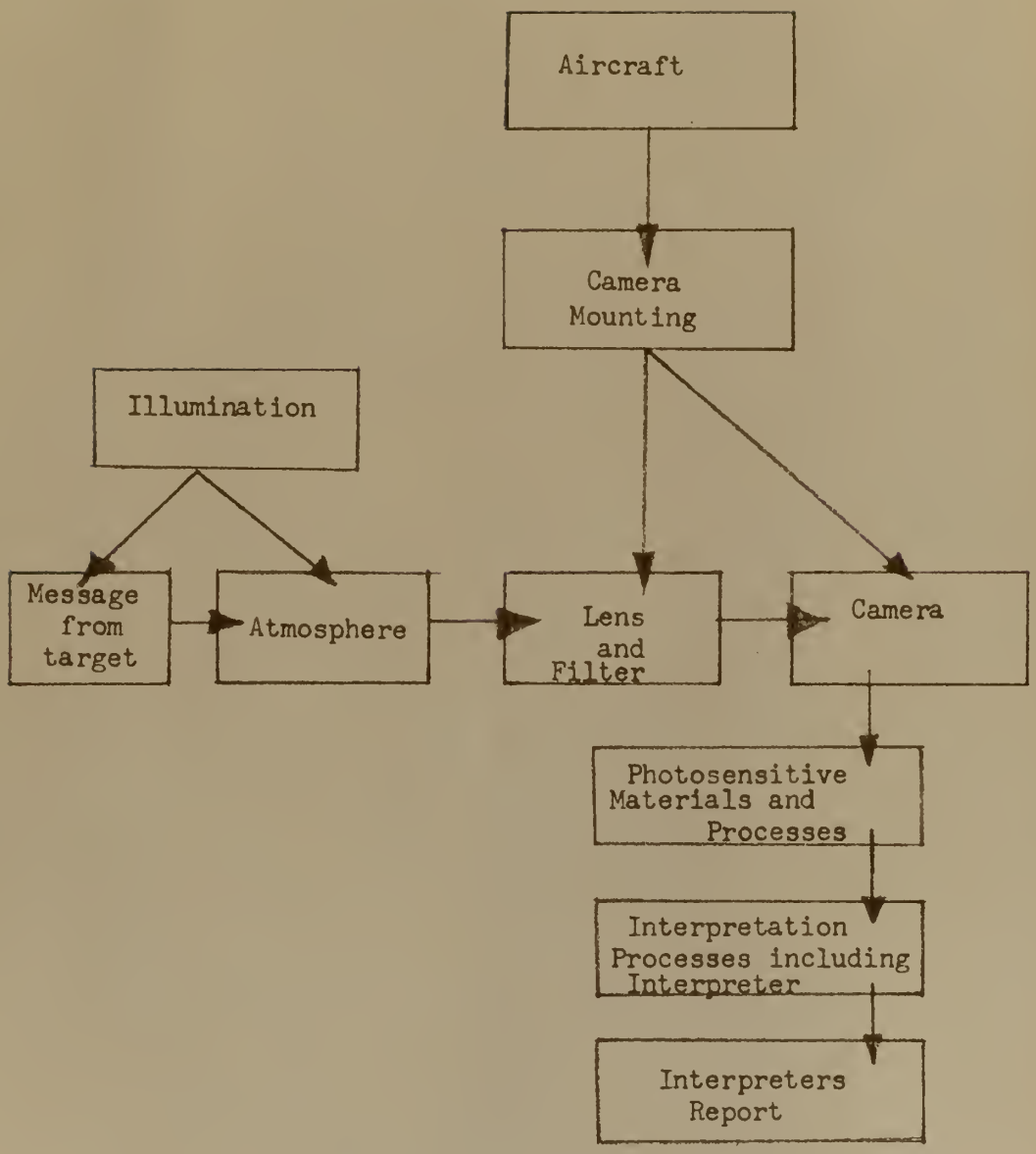


Illustration 14

The Aerial
Photographic System
From Boston University O.R.L. Technical Note No. 85

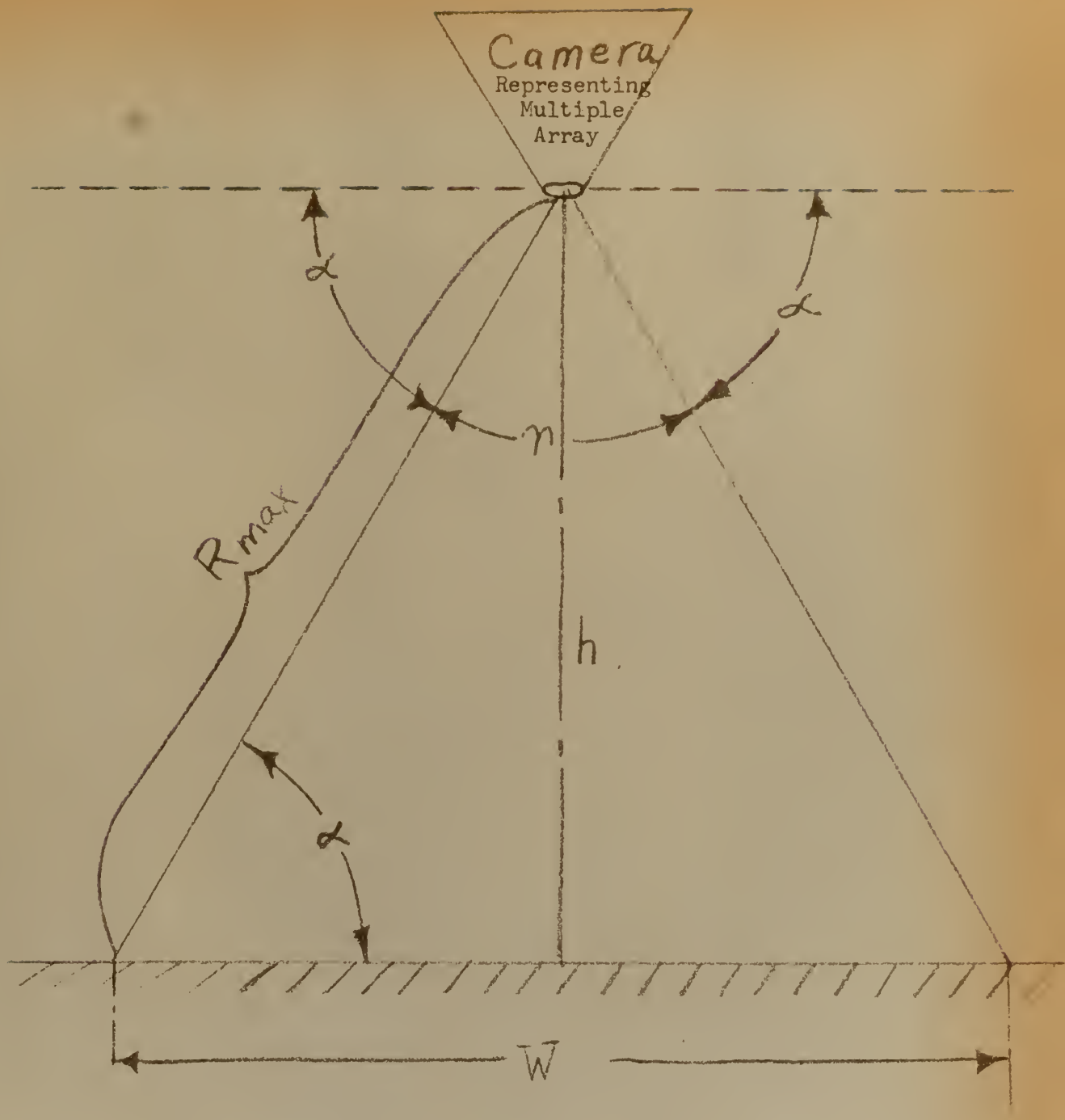


Illustration 15

Sweep Width of Daytime
Aerial Photography

be calculated may be limited in one of two ways. It may be limited by the target contrast and perimeter as in the daylight case if illumination is sufficient or by illumination of the target area.

Both limiting conditions may be characterized by values of R_{max} of which the minimum will be the value used in the computation of W_{np}, night photographic sweep width.

Let us consider that we have a photographic aircraft flying as in Illustration 16.

We first define the following factors*:

h = flying altitude

R_{max} = air distance to extreme of coverage

α = dip angle of coverage

γ = coverage angle

L_s = strength of source in candles

Σ = required film exposure in meter candle seconds

$\Sigma = Et = E_n t_o = E t_e = \frac{1}{s}$

E = illuminance on emulsion in meter candles

t_o = length of time shutter is open

$t_o = nt_o$ effective exposure time

η = optical efficiency of shutter

B = luminance of object in candlesper

τ = transmittance of lense

F = f number of lense stop

S = A S A exposure index of emulsion

We now proceed as follows:

(1) Calculate B, luminance of target produced by source L_s.

* Photographic Optics by Greenleaf --Macmillan. Chapters 13 and 15.

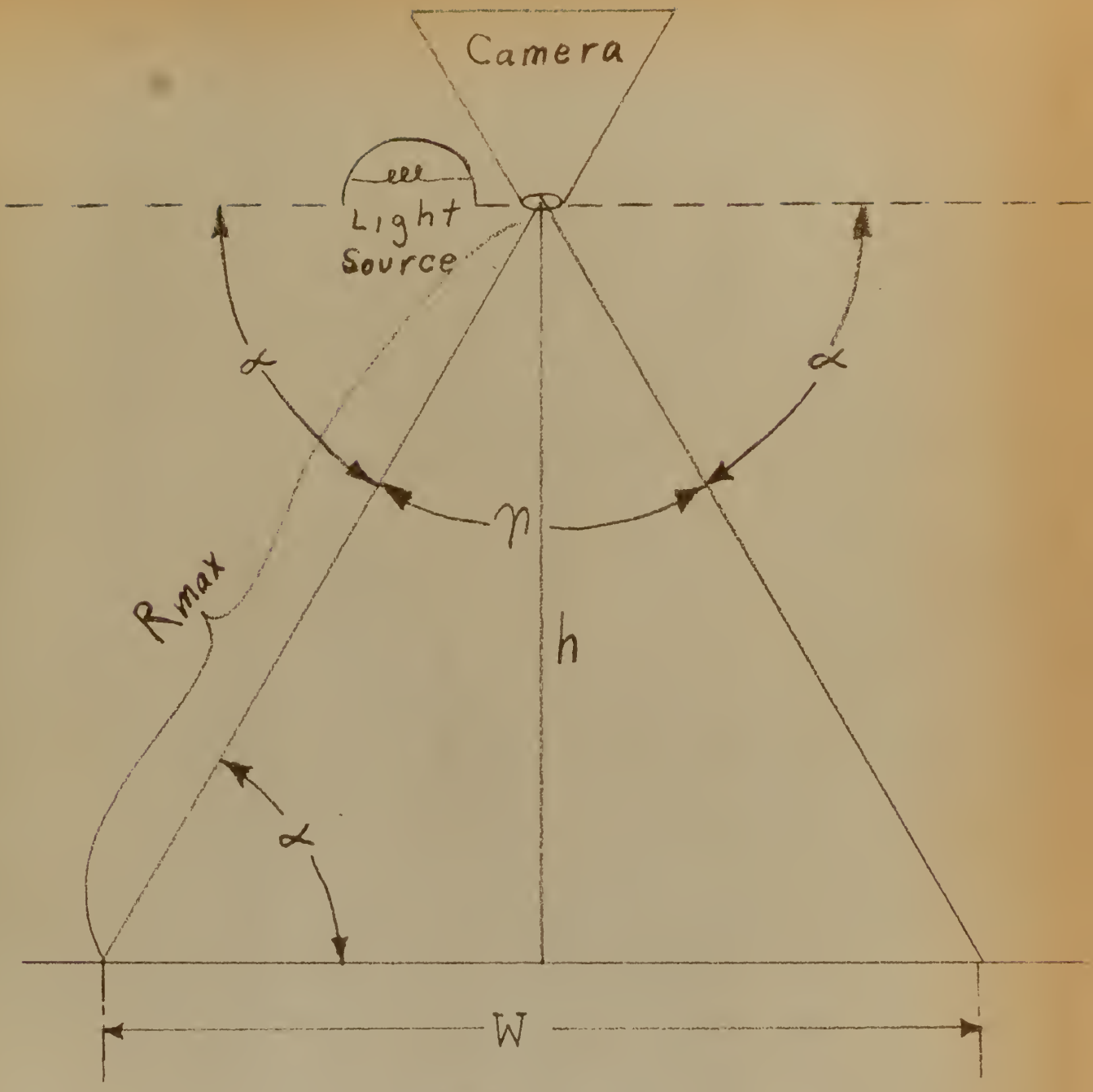


Illustration 16

Sweep Width of Night
Aerial Photography

$B = \frac{L_s}{R^2_{max}}$ luminance in candles/m^2

(2) Calculate illuminance on emulsion from target meter candles

$$E = \frac{\pi B \tau}{4F^2} = \frac{\pi L_s \tau}{4F^2 R^2_{max}} \quad \text{meter Candles}$$

(3) Calculate requirer film exposure

$$\sum = Et = E \eta t_o$$

(4) Solving for t_o and substituting

$$t_o = \frac{\sum}{E\eta} = \frac{4F^2 R^2_{max}}{\eta \pi L_s \tau S}$$

(5) Solving for R_{max}

$$R_{max} = \sqrt{\frac{t_o \eta \pi L_s \tau}{4F^2}}$$

Now we have a value of R_{max} in terms of t_o, the time the camera shutter is open; L_s the illumination source strength and other system characteristics. By increasing t_o or L_s we can increase R_{max}. By decreasing F we can also increase R.

Each of these factors has a limit beyond which it cannot be increased or cannot economically be increased. t_o is limited by the image motion compensation mechanism. If u is the velocity of a target image on the camera plate, a function of aircraft speed and focal length, and Δ p is the travel of the image motion compensation mechanism then

$$M_{ax}\, t_o = \frac{\Delta p}{u}$$

L_s is limited by the size and energy requirements of light sources of increasing size. Practical limits are reached at some maximum L_s. F is limited by the lens itself, the value of R_{max} can be increased by

decreasing F until the entire lens is transmitting light.

The value of R_{max} computed here may be greater or less than R_{max} as determined for the daylight case.

$$R_{max\ (night)} = \text{Min} \begin{cases} R_{max} \text{ from part a} \\ R_{max} \text{ from part d} \end{cases}$$

then $W_{nightphoto} = 2\ R_{max\ (night)} \cos \alpha$

S_L night is then calculated as was the day photography sweep rate.

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